Generating and Photographing Laminar, Circular Vortex Rings in Air

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Abstract

A vortex ring generator and design of a photographic experiment to capture the propagation of circular vortex rings in air are described. Photographic methods are employed to approximate vortex ring size, velocity and circulation. Smoke is used as a seed particle to visualizing the bulk motion and internal structure of circular vortex rings and the resulting image is discussed in light of the well-established physics surrounding vortex ring propagation and development; namely Biot-Savart Law, Helmholtz's theorems and vortex ring instability.

Objective

To design an experimental setup that generates vortex rings in air with a reliable method for visualization. Furthermore, to design a photographic setup capable of capturing the dynamics of the propagation of vortex rings in quiescent air.

Background

<u>Vortex Ring Description</u>

A vortex ring is fundamentally a vortex line which is closed into the shape of a circle. This geometry is in accordance with Helmholtz's third vortex theorem which states that a vortex line cannot end in a fluid, it must either end at a boundary or be closed in shape [1]. The action of the vortex line creates a toroid of fluid wrapped around the vortex line which travels with it in the direction of the ring axis.

<u>Vortex Ring Formation</u>

Although the propagation of a laminar vortex ring can be approximated by potential flow theory (inviscid dynamics) the formation of a vortex ring is a viscous process. Viscosity is necessary to generate a vortex ring because vorticity can neither be created nor destroyed in a barotropic, conservative body force fluid that is inviscid [1]. When a slug of fluid impinges on a stationary fluid the sides of the fluid column form a vortex sheet between the two fluids [2]. The sides of the fluid slug are slowed down by viscosity and roll up around the sides of the fluid column. This action generates the circulation which will be carried by and propel the vortex ring.

<u>Vortex Ring Propagation</u>

After formation, vortex rings propagate under the action of their own self-induced velocity. A vortex line induces a velocity on neighboring fluid in a direction orthogonal to the vortex line. The magnitude of this induced velocity depends on the distance of separation between the vortex line and a point of interest, the orientation of the point of interest relative to the vortex line and the circulation of the vortex line. These qualities are related by the Biot-Savart Law where **V** is velocity, Γ is circulation, **dl** is the vortex line segment and **r** is the vector separating the point of interest and the line segment **dl**:

$$\mathbf{dV} = \frac{\Gamma}{4\pi}\frac{\boldsymbol{dl} \times \boldsymbol{r}}{|\boldsymbol{r}|^3}$$

In addition to inducing a velocity in the surrounding fluid a circular vortex line similarly induces a velocity on itself. To be more precise each line segment of a vortex line induces a velocity on all points in the vortex ring which are not in alignment with the line segment. Helmholtz's second vortex theorem states that the strength of a vortex line is constant along its length [1]. Since every point on a circle is an equal distance from the rest of the circle and since the circulation is constant around a vortex ring all points on a vortex ring propagate at the same speed. This induced velocity causes a circular vortex ring to travel, undeformed, in the direction of the axis of the ring. This phenomenon is self-sustaining due to Helmholtz's first vortex theorem which states that the vortex line moves with the fluid. Helmholtz's fourth vortex theorem states that the circulation of a vortex line remains constant with time [1]. Therefore, a laminar vortex ring in an inviscid, quiescent, infinite fluid propagates at a constant speed, indefinitely.

Vortex Ring Transition

Although the inviscid flow approach used in the previous section to describe vortex ring propagation is reasonable for a laminar vortex ring in reality no fluid is invisicid and laminar vortex rings do not remain so forever. Given sufficient time a laminar vortex ring will develop wave-like, azimuthal instabilities (sometimes referred to as the Widnall instability) which distort the shape of the vortex ring and grow with time. When these instabilities become sufficiently large they break down and the vortex ring will transition to turbulence [3]. Turbulent vortex rings have less orderly structures and tend to lose fluid from the toroid due to mixing, which in the case of flow visualization manifests as seed particles deposited in the ring's wake.

Method

The following section outlines the experimental set-up designed to capture photographs of circular vortex rings propagating n air.

Control of Environment

The doors, windows and vents in the room where the experiment was conducted were shut, and sealed where necessary, to create a test environment isolated from external light sources and drafts/air currents. The experiments were not started until the air in the room had settled from experimenter motion during set up. While conducting the experiment the experimenter wore a mask to prevent exhaled breathes from introducing currents into the air which the vortex rings would pass through. A black fabric photographic background was hung behind the intended path of the vortex rings to eliminate distracting background elements in the experimental photos.

Lighting

Lighting came from two manmade sources, a low output trigger light and a high output off-camera flash. The first source was a 24 LED array with a relatively low 50 lumen output. The 24 LED array was placed in a box with a slot cut in the top such that a directional narrow sheet of light was projected upwards through the path of the vortex rings but not onto the background or walls of the room. In this orientation the beam of light was orthogonal to both the axis of vortex ring propagation and to the axis line of the camera lens barrel. This light provided enough illumination for the experimenter to track the vortex ring motion in the dark room and by placing it just upstream of the image frame it provided a trigger event for actuation the camera shutter.

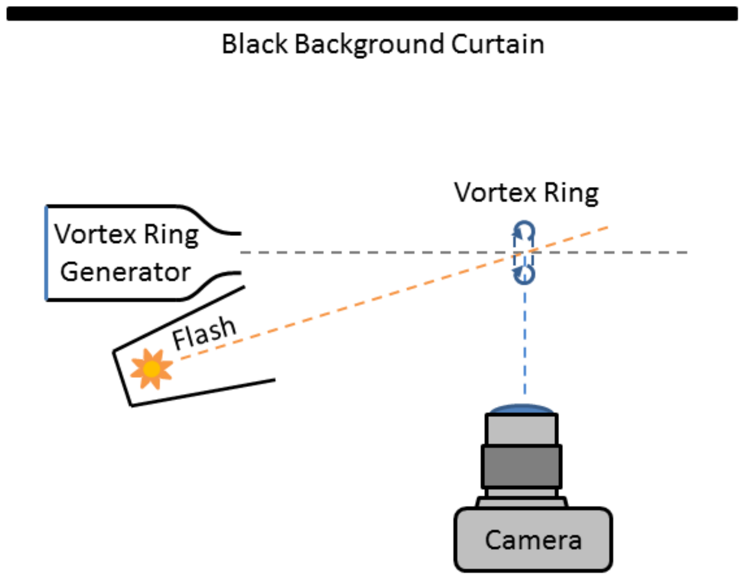

Figure 1: Experimental Setup

A single flash was mounted off-camera on a separate tripod and attached to the camera with a 10 foot off-camera shoe cord. The flash was placed adjacent to the vortex ring generator at a slight angle askew of the ring propagation axis pointed in the direction of propagation (Figure 1). A housing lined with reflective foil was wrapped around the flash head to ensure a highly directional and focused column of light was formed without any diffuse reflections off of other objects in the room. The flash was used in high speed sync mode to enable shutter speeds in excess of 1/250s. Flash output was set manually to ensure more repeatable light output than would be possible with the camera TTL (Through The Lens) system. The beam zoom of the flash was set to automatically track the focal length of the lens. No gels or diffusers were attached to the flash and so the output beam was a strong, directional light with a slightly cool white color. The flash was triggered by the camera.

Camera

A full-frame Canon 6D D-SLR (Digital Single Lens Reflex) camera was used for imaging the vortex rings. The camera was mounted on a tripod and oriented such that the lens barrel axis was perpendicular to the vortex ring propagation axis and with the imaging plane ~0.5m from the vortex ring centerline (Figure 1). Focus was set using the camera's auto focus system to lock on to a target image printed on a card which was placed in line with where the vortex rings would pass. Once focus had been achieved the target card was removed and the auto focus system was switched off. Photographs were taken with the camera in manual mode. Shutter speed was set to 1/1000s to freeze the motion of the vortex rings and also to provide too short of an exposure time for the remaining ambient light and diffusely reflected trigger light to expose the background. Aperture was set to f/10 to allow for a sufficiently broad depth of field to resolve the vortex rings. Sensitivity was set to ISO 500 which provided enough light to see the vortex rings in the images but was low enough to avoid excessive grains in the relatively dark images. These settings would typically allow in too little light for such a dark shooting environment but were enabled by setting the flash to almost maximum light output. The photographs were written to a RAW sRGB format and post processed off camera.

Optics

A 24-70mm f/2.8 lens was used for imaging. These lenses use Ultra-low Dispersion (UD) glass and are known to produce particularly sharp photos. For this image the zoom was set to 50mm. The lens hood was attached to prevent lens flare as different positions of the flash and camera were experimented with. No filters or additional optical elements were attached to the lens. Aperture was controlled by the camera body, as is typical with modern DSLR equipment.

Vortex Ring Generator

Vortex rings were generated with a specially designed devise which consisted of a 1.74L cavity with an 8x8cm flexible membrane on one end and a ~3.5:1 contraction down to a 3cm diameter circular orifice on the other end (Figure 2). Into the cavity was inserted a smoldering incense stick. The smoke produced by the incense stick has particle size on the order of 0.1-1µm [4]. A vortex ring was produced by depressing the flexible member into the cavity, reducing its volume, and forcing a slug of fluid out of the orifice.

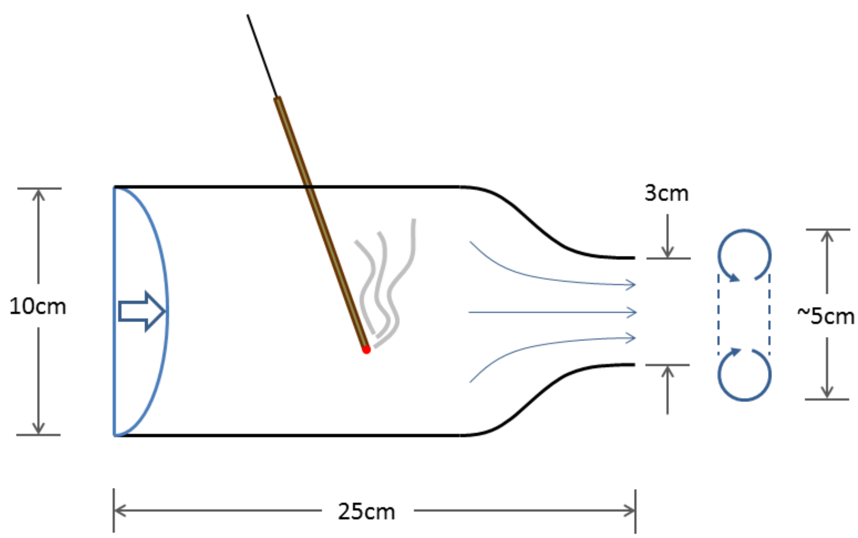

Figure 2: Vortex Ring Generator

Results

Prior to capturing frozen motion images the vortex ring velocity was experimented with by varying the displacement rate of the vortex generator membrane. Rings that started with high initial velocities developed multi-lobed, azimuthal instabilities, transitioned to turbulence and typically lost structural coherence shortly thereafter. Rings with low initial velocities traveled a short distance before losing all forward momentum after which they sat suspended in air with no visible circulation until the smoke core diffused or mixed into the ambient air. Rings with a moderate initial velocity propagated the longest distances as a coherent vortex ring before being destroyed. The moderate initial velocity was used for subsequent experimentation because of the ring's longevity and laminar flow pattern.

To quantify the velocity of the vortex rings a set of ten representative vortex rings were photographed with a longer (1/3 sec) shutter speed. Using the distance traveled by the core of the vortex ring during the exposure (measured in pixels) and the calibrated width of the image in the plane of the vortex ring (17cm) a velocity of each ring was computed. The velocity of the vortex rings varied from 0.173m/s to 0.234m/s and averaged 0.202m/s (Table 1).

Ring #	Pixels Traveled	Fraction of frame	Distance Traveled (m)	Velocity (m/s)
1	2358	0.43	0.073	0.220
2	2388	0.44	0.074	0.223
3	2052	0.38	0.064	0.191
4	2472	0.45	0.077	0.230
5	1998	0.37	0.062	0.186
6	2513	0.46	0.078	0.234
7	1854	0.34	0.058	0.173
8	1896	0.35	0.059	0.177
9	2046	0.37	0.064	0.191
10	2082	0.38	0.065	0.194
			Minimum	0.173
			Maximum	0.234
			Average	0.202
			Stnd Dev.	0.023

Table 1: Vortex Ring Velocity Computation

Using a similar process the total diameter of the vortex rings produced by the vortex ring generator was approximated to be 5.1cm. For this size and the average velocity Reynolds number is ~690. Measuring from the core of the vortex on opposite sides of the ring the diameter of the circular vortex line was found to be roughly 3.9cm. Using the ring diameter the Biot-Savart law was integrated numerically to find the induced velocity at a point on the ring as a function of circulation. This result is:

$V=11.838\Gamma$

With an average velocity of 0.202m/s the circulation of the typical vortex ring was therefore 0.0175m^2/s.

A frozen motion image of two vortex rings produced by the vortex ring generator with moderate initial velocity was photographed with the previously described experimental set up. The resulting image is included in Figure 3.

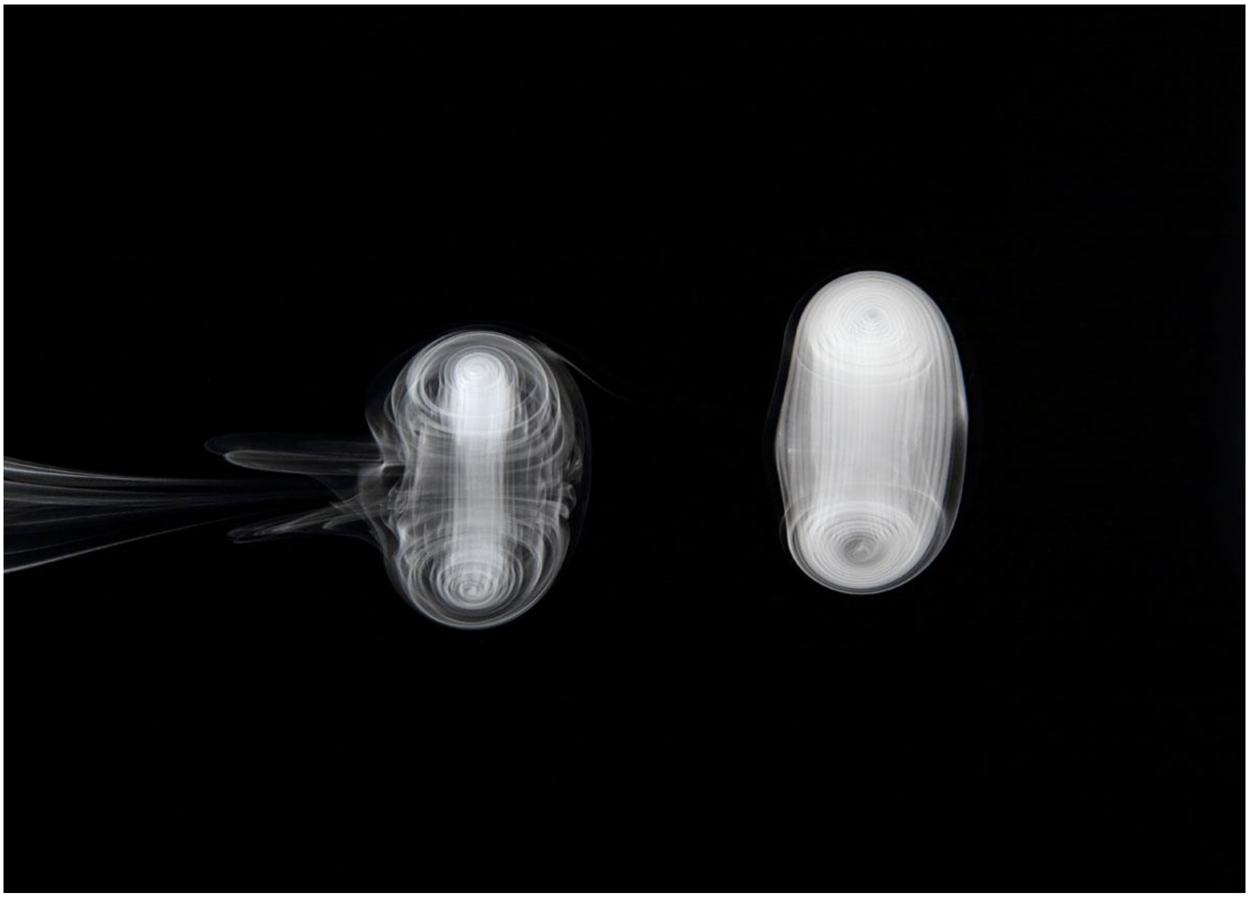

Figure 3: Photo of two circular vortex rings propagating left to right seeded with smoke

Focal Length	50mm
Shutter Speed	1/1000s
Aperture	f/10
ISO	500
Autofocus	Off
White balance	Auto
Gamut	sRGB
Format	RAW
Pixels WxH	5107x3648
Field of View WxH	21cmx15cm

Table 2: Photo Settings

The settings used in this photo are documented in Table 2. The RAW photo required only limited post processing. The image was first cropped to adjust the framing. Brightness was increased along with contrast. Finally clone stamp was applied to a few places in the image to remove errant dust particles which were illuminated by the flash. See Appendix A for unedited image.

Discussion

In the image (Figure 3) two vortex rings are seen propagating left to right across the frame. The left frame of the image is roughly 3-5cm to the right of the vortex generator orifice. The more mature of the two rings (right ring) has a higher density of smoke seeding and therefore appears brighter in the image. Behind the left ring remnants of the slug of fluid which formed it are visible in the form of a smoke tail leading back to the vortex ring generator orifice. The right ring has propagated away from its tail into the unseeded stationary fluid surrounding the vortex ring generator. It appears that the tail of the right ring has been rolled up into the outer layer of the left vortex ring and a trace of it is still visible as a spur on the upper right side of the left ring.

The seeded flow in the vortex ring reveals an interesting characteristic of the ring structure. Although vortex rings are often thought of as a closed toroid it is apparent the ring is actually a toroidal spiral [5]. As fluid rotates around the vortex line the seeded fluid in the ring is wound in to ever tighter spirals. In the more densely seeded right ring between 10 to 12 layers are apparent in the cross section.

The distance of separation between the two rings in the image is roughly 1.75 diameters. At this distance the self-induced velocity for each ring will be larger than the inter-ring induced velocity. However, the inter-ring induced velocity will not be negligible. If the rings were coaxial (some axial misalignment is apparent) and traveling at similar velocities the right most ring would be induced to grow in diameter and the left ring would shrink. As the right ring grew in diameter its velocity would drop and conversely the left ring would speed up. This process would progress until the left ring passed through the center of the right ring at which time the process would repeat with the roles reversed, until transition destroyed them. In the current circumstance, however, the axial misalignment caused the rings to collide and both were subsequently destroyed.

Conclusions

The vortex ring generator designed for this experiment was found to be effective in producing circular vortex rings with sufficiently dense smoke seeding to allow the internal structure of a propagating vortex ring to the revealed. Furthermore, the experimental set up allowed the vortex ring motion to be frozen in the image due to proper use of lightning and shutter speed. The control of the direction of the light of the flash and the elimination of the majority of ambient light coupled with the use of the black backdrop effectively isolated the illuminated vortex ring from the background. By using long exposures the motion blur of the passing vortex rings was used to compute an average velocity of 0.202m/s for the ring which corresponds to a circulation of Γ=0.0175m^2/s for a 3.9cm core-to-core vortex ring diameter. The smoke seeding, in addition to revealing the bulk motion of the ring, revealed the internal structure of the ring which is a toroidal spiral.

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Appendix A: Unedited image