Inverse Forest

(Featuring Saffman-Taylor and Rayleigh-Taylor Instabilities)

Flow Visualization Team#2 Project -2

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Final Picture:

Fig: 1 **'Inverse Forest'** - Final picture portraying Saffman-Taylor and Rayleigh-Taylor instabilities (and many other things).

1. Introduction:

The image 'Inverse Forest' was submitted as a part of coursework for Flow visualization Group project -2. The basis of the project is visualization of the Saffman-Taylor and Rayleigh-Taylor instabilities.

The project started with the motive of experimenting instabilities in a Hele-Shaw cell with ferrofluid under influence of magnetism. The experiment was unsuccessful as the ferrofluid film was so thin that magnetism had no visible effect on it. The experiment was hence continued without using a magnetic field.

2. Physical phenomena:

2.1 Hele-Shaw flow:

Hele-Shaw is Stokes flow (low Reynolds number Re<<1) between two parallel flat plates separated by an infinitesimally small gap (micro-flow).The governing equation of Hele-Shaw flows is identical to that of the inviscid potential flow and Darcy's law. It thus permits visualization of this kind of flow in two dimensions.

$$\mathbf{u} = \nabla p \frac{z^2 - H^2}{2\mu}$$

Where,

u = velocity, m/s

$p(x, y, t)$ = local pressure, N/m^2

μ = Dynamic fluid viscosity, N.s/m^2

This relation and the uniformity of the pressure in the narrow direction z permits us to integrate the velocity with regard to z and thus to consider an effective velocity field in only the two dimensions x and y. When substituting this equation into the continuity equation and integrating over z we obtain the governing equation of Hele-Shaw flows,

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0.$$

This equation is supplemented by the no-penetration boundary conditions on the side walls of the geometry,

$$\nabla p \cdot \hat{n} = 0$$

Where $\hat{n}$ is the unit vector perpendicular to the side wall.

2.2 Saffman Taylor Instability (Viscous Fingering) [2] [3]:

Viscous fingering is the formation of patterns in a morphologically unstable interface between two fluids in a porous medium or in a Hele-Shaw cell. It occurs when a less viscous fluid is injected displacing a more viscous one. In a natural environment it mostly occurs by gravity if the interface is horizontal separating two fluids of different densities, being the heavier one above the other. In the rectangular configuration the system evolves until a single Saffman–Taylor finger forms. In the radial configuration the pattern grows forming fingers by successive tip-splitting. The picture is a part of a radial system involving gravity in a Hele-Shaw cell. The fluids interacting in this case are the denser ferrofluid and air at room conditions.

The mathematical description of viscous fingering is the Darcy's law for the flow in the bulk of each fluid, and a boundary condition at the interface accounting for surface tension.

The Darcy's law is given by

$$q = \frac{-k}{\mu}\nabla P$$

Where,

q = Discharge per unit area, m/s

k = Intrinsic permeability of the medium, m^2

∇P = Pressure gradient vector, N/m^3

μ = Dynamic viscosity, $N.s/m^2$

2.3 Rayleigh Taylor (RT) Instability

The Rayleigh–Taylor instability, is an instability of an interface between two fluids of different densities which occurs when the lighter fluid is pushing the heavier fluid. [1] If a parcel of heavier fluid is displaced downward with an equal volume of lighter fluid displaced upwards, the potential energy of the configuration is lower than the initial state. Thus the disturbance will grow and lead to a further release of potential energy, as the more dense material moves down under the effective gravitational field, and the less dense material is further displaced upwards. As the RT instability develops, the initial perturbations progress from a linear growth phase into a non-linear or exponential growth phase, eventually developing plumes flowing upwards due to gravitational buoyancy and spikes falling downwards. In general, the density disparity between the fluids determines the structure of the subsequent non-linear RT instability flows (assuming other variables such as surface tension and viscosity are negligible here and are accounted in Saffman Taylor instability). The difference in the fluid densities divided by their sum is defined as the dimensionless Atwood number, A. [4] For A close to 0, RT instability flows take the form of symmetric fingers of fluid; for A close to 1, the much lighter fluid "below" the heavier fluid takes the form of larger bubble-like plumes.[1]

$$\mathrm{A} = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$$

Where,

A = Atwood number, Dimensionless

ρ_1 = density of heavier fluid, kg/m^3

ρ_2 = density of lighter fluid, kg/m^3

3. Visualization:

The visualization for the Inverse Forest effect was done essentially using the closely spaced parallel glass plates of a typical Hele-Shaw setup. The other essentials used in the experiment are

1. 10 ml Ferrofluid as the denser fluid.

2. White acrylic sheet as backdrop and for diffusing light.
3. 2 X 9 LED array backlight
4. Camera with tripod

Few drops of ferrofluid was added to the base glass plate and covered with the top glass plate. This creates a thin film of ferrofluid between the plates. When the plates are separated from one side, air tries to fill in the void created by downward movement of ferrofluid due to gravity. This provides a suitable environment for air-fluid interaction and all the phenomena discussed above gives rise to the patterns captured in the picture.

The Hele Shaw cell and illumination were sourced from Prof. Jean Heartzberg's Flow visualization lab at CU Boulder. The experiment was conducted at the Integrated Teaching and Learning Laboratory at CU Boulder.

The LED illumination was used to give a neutral color and to avoid radiative heat input to the system which happens in case of incandescent lighting. The LED array was placed below the acrylic sheet that diffused the incoming light giving an illuminant white background. The fluorescent room lights in the ceiling were also used to provide balanced light on both sides of the plate.

The approximate height used during visualization was 0.5 meter above the top plate surface at an angle of 30^o from the normal to the surface. A closed room was used with an ambient temperature around 25^oC and there were no other disturbances.

The camera was focused on the fluid and a wireless infrared shutter trigger was used.

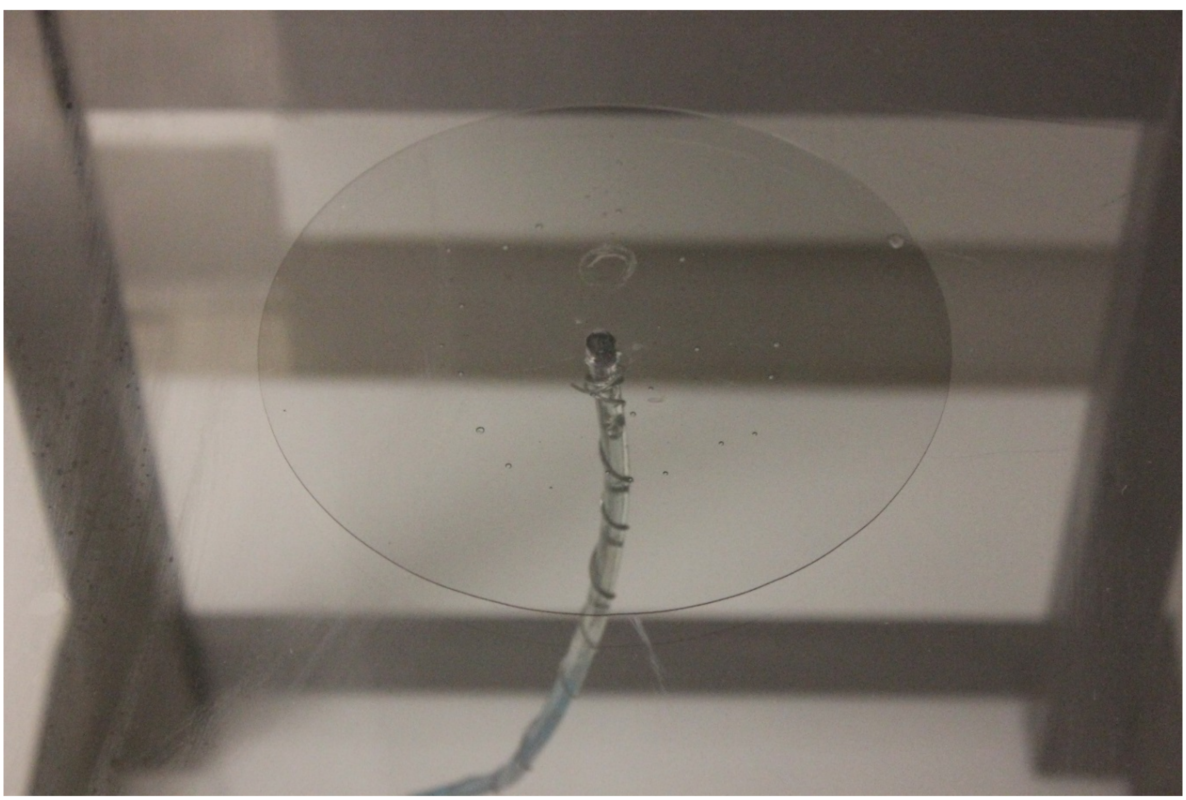

Fig 2: **Initial condition in a Hele Shaw cell with corn syrup.**

4. Photography:

Details of the original photograph are as follows

File Name	IMG_1438.CR2
Camera Model	Canon EOS REBEL T3i
Firmware	Firmware Version 1.0.2
Shooting Date/Time	3/14/2014 12:52:16 PM
Owner's Name	
Shooting Mode	Manual Exposure
Tv(Shutter Speed)	1/80
Av(Aperture Value)	7.1
Metering Mode	Evaluative Metering
ISO Speed	800
Auto ISO Speed	OFF
Lens	EF-S18-55mm f/3.5-5.6 IS II
Focal Length	55.0mm
Image Size	5184x3456
Aspect ratio	3:2
Image Quality	RAW
Flash	Off
FE lock	OFF
White Balance Mode	Auto
AF Mode	Manual focusing
AF area select mode	Manual selection
Picture Style	Auto
Sharpness	3
Contrast	0
Saturation	0
Color tone	0
Color Space	sRGB
Long exposure noise reduction	0:Off
High ISO speed noise reduction	0:Standard
Highlight tone priority	0:Disable
Auto Lighting Optimizer	Standard
Peripheral illumination correction	Enable
Dust Delete Data	No
File Size	24769KB
Drive Mode	Self-Timer Operation
Live View Shooting	ON
Camera Body No.	362077139256

5. **Post Processing:**

Post processing was done using Adobe Photoshop CS6, licensed to University of Colorado, Boulder.

The picture resolution was edited from 5184 X 3456 to 3761 X 1740 pixels to crop out unnecessary details such as the central hole and stagnant fluid.

Dark spots on the original picture were cleared using duplicate layers. 24 such instances were done to get the final picture.

6. **Perspective and the Inverse Forest Theme:**

The final picture 'Inverse Forest' generates a wide range of thought or imagination in the mind of the observer depending on their perception or state of mind. However, as a creator, the mind perspective while making the picture is described. This would also explain the choice of colors, camera settings and post processing choices made. The orange shades below the dark forests resembles a morning/evening sun. By normal convention the orange sky shades appear above a forest but in this case it appears below.

The picture can be related to 'Expressionism' classification of art and various surreal themes can be imagined like the forest burning from below etc.

7. **End note:**

Though the picture does a good work in describing Saffman-Taylor and Rayleigh-Taylor instabilities, there is still scope for improvement. The top two corners of the final image are unclear but are not cropped because cropping made it look unreal as no matching layers could be substituted. In future attempts, this problem can be taken care of by maintaining a clean edge surface. For intense fluids like ferrofluids used in a setup and experiment as conducted in this case, the center hole can be blocked to avoid ferrofluid spillage on the acrylic sheet which is hard to clear and gives out a bad stained picture as it blocks light. Replacement of the glass plates can be considered to save editing time as the small pits in the glass cause dye accumulation thus resulting in dark spots.

8. **References:**

[1] Sharp, D.H. (1984). "An Overview of Rayleigh-Taylor Instability"

[2] Viscous fingering in Hele-Shaw cell By P. G. SAFFMAN - Applied Mathematics, 217-50, California Institute of Technology, Pasadena, CA 91 125, USA.

[3] The Penetration of a Fluid into a Porous Medium or Hele-Shaw Cell Containing a More Viscous Liquid - P. G. Saffman and Geoffrey Taylor - Published 24 June 1958 doi:10.1098/rspa.1958.0085Proc. R. Soc. Lond. A 24 June 1958 vol. 245 no. 1242 312-329

[4] Glimm, J.; Grove, J. W.; Li, X.-L.; Oh, W.; Sharp, D. H. (2001). "A critical analysis of Rayleigh–Taylor growth rates". J. Comput. Phys. 169 (2): 652–677. Bibcode:2001JCoPh.169..652G. doi:10.1006/jcph.2000.6590.