

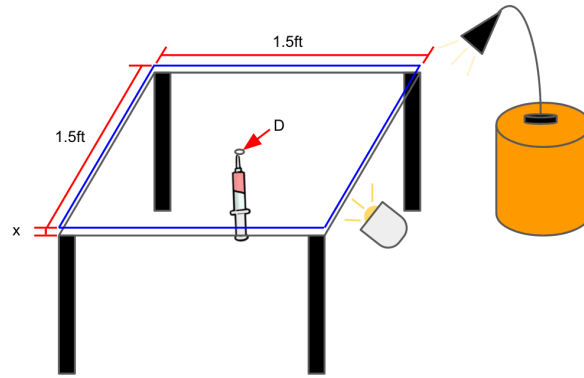
Final Group Project 2 Image: Saffman-Taylor instability formed by liquid detergent and red india ink

1 Purpose

This visualization is a part of the second Flow Visualization group project, in which the Hele-Shaw cell was used. The purpose of this visualization was to capture the Saffman-Taylor instability that this setup produces, by first experimenting with different combinations of fluids with varying viscosities. The final image that was captured, shown above, was a combination of Tide liquid washing detergent and red india ink. This photo was achieved thanks to the help of my teammate, Gardner Nichols, who injected these fluids into the Hele-Shaw cell while I photographed the results.

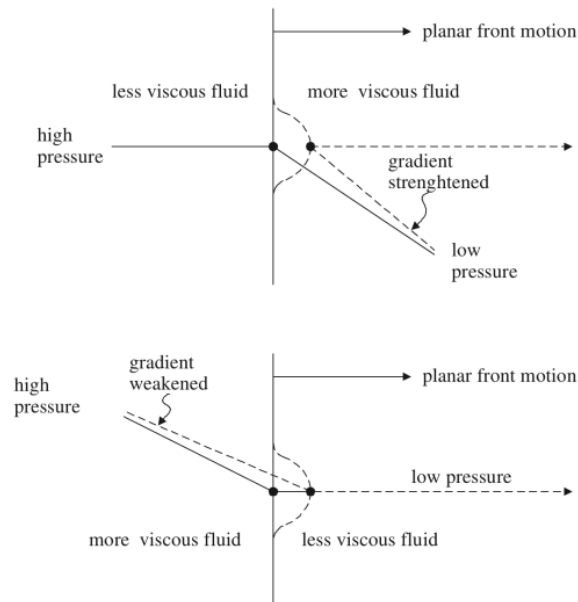
2 Flow Apparatus

The apparatus used to achieve the visualization is known as a Hele-Shaw cell, named for the type of flow with the same name that describes a flow between two parallel flat plates separated by an infinitesimally small gap. The physical setup consisted of an acrylic sheet 1.5'x1.5' in size with a 1/16" diameter hole D at the center, and a single sheet of glass of the same size on top of it. The two flat surfaces are placed on top of a stand so that the hole on the acrylic sheet can be accessed from below to introduce liquids into the system. The gap between the two surfaces is approximately $x = 0.75\text{mm}$. A syringe was used to inject fluids into the cell, and one light was above and below the system. An image of the apparatus is shown below. Since the hole is at the center of the sheet, this is known as a "radial source flow" Saffman-Taylor instability. [2] Other experiments have utilized gravity by orienting the Hele-Shaw cell perpendicular to the ground, but this cell is parallel to the ground and is not as affected by a gravitational force.



Hele-Shaw cell setup

As mentioned previously, several combinations of fluids were attempted to create this visualization, but liquid washing detergent and india ink were the two fluids used to demonstrate the Saffman-Taylor instability in the final image. As discussed by [1], the instability arises, “when two fluids of different viscosity are pushed by a pressure gradient through a Hele-Shaw cell or allowed to drain through such a cell under their own weight.” The viscosities of the fluids, although not published values, can be compared to other fluids with similar properties. The viscosity of the liquid detergent can be approximated as that of corn syrup, with a value of $1.38 \text{ Pa}\cdot\text{s}$ and the ink is similar to water but may have a slightly higher value, which has a viscosity of $8.94E - 04 \text{ Pa}\cdot\text{s}$. The two fluids having different viscosities interact to form “fingers” when the more viscous fluid is injected first, and the less viscous fluid afterward which then displaces the first fluid. As discussed by [3], this instability results from the combination of the low viscosity fluid entering the high viscosity fluid at a high pressure, where the pressure gradient increases as this contact occurs. The pressure gradient is in the higher viscosity fluid to begin with, and so it supports the increase of the gradient as the low viscosity fluid is injected into it at a high pressure. This is shown in the top of the image shown in the figure below. In the opposite case, where the high viscosity fluid enters the low viscosity fluid with a high pressure, the gradient tends to weaken since there is little resistance when it comes into contact with the fluid.



Saffman-Taylor explanation from [3], showing a high pressure disturbance introduced into a high viscosity fluid from a low one (top), and into a low viscosity fluid from a high one (bottom)

In the case of the bottom diagram shown in the figure above, the introduction of the less viscous fluid first will not cause these instabilities to form. Rather, they will flow radially with no instability at the boundary. A demonstration of this is shown in the image below where the india ink was injected into milk, with the ink having a higher viscosity than the milk. The results were significantly less interesting than what could be achieved if they were introduced in an opposite manner, and we soon changed the liquids with which we were working.



India ink (high viscosity) introduced into Hele-Shaw cell containing milk (low viscosity) with some exciting results

2.1 Process

The detergent was injected first since it has a larger viscosity, and then the red india ink was introduced into the system afterward. This formed a traditional Saffman-Taylor instability with large bulbous fingers spreading out with the injection of the india ink. After all of the ink was injected into the system, the syringe was removed from the hole in the acrylic sheet, and the two liquids were allowed to drain from the hole under their own weight. After about 1 minute, this draining caused the fingers to decrease in size and become more sharp-looking fluid structures. The pressure gradient which was initially directed radially outward from the hole was reversed as soon as the syringe was removed from the apparatus. The open hole allowed for fluid to escape from the center, so the direction of the pressure gradient was then directed radially inward toward the hole.

It should be noted that the asymmetry present in the system is due to the contact of the two surfaces, where there is a smaller gap near the top of the final image, and the gap is slightly larger where the fingers occur. This explains why the fingers aren't evenly distributed radially in the cell.

3 Visualization Technique

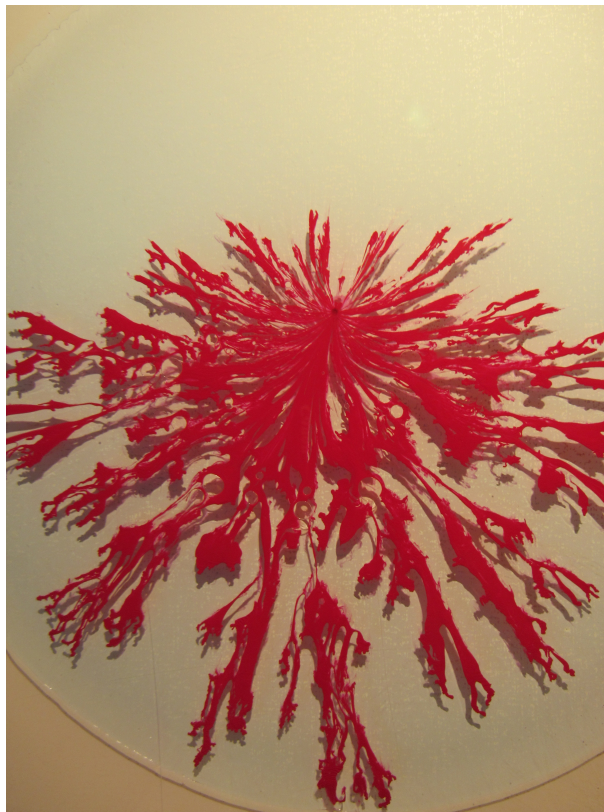
The technique used to capture this image is known as the seeded boundary method. Since the acrylic sheet has one side that is white, it creates the background against which the Saffman-Taylor instability can be visualized. The red ink has a bold, consistent color which creates a significant contrast for the instability to be clearly shown. Two lights were used to illuminate the instability, while all other lights in the surrounding area were turned off. One light was a spotlight lamp with a 4.5 Watt LED which produces 200 lumens and was placed on top of a 5-gallon bucket and directed toward the top surface of the Hele-Shaw cell. The equivalent warmth for this light is 2700 Kelvin, which gives off a bright and relatively warm color. The other light was a 40 watt soft white incandescent bulb, which produces 490 lumens and also has an equivalent warmth of 2700 Kelvin. This light was placed underneath the cell to provide a backlight for the visualization. The approximate locations of the lighting used can be found in the diagram of the Hele-Shaw cell setup that was shown previously.

4 Photographic Techniques

The surface of the Hele-Shaw cell was approximately 6 inches away from the camera lens, while the focal length was 4.3mm. The digital camera that was used to perform this visualization is a Canon PowerShot SX520 HS, with a focal length of 4.3-180.6mm, aperture 1:3.4-6.0. The settings used to capture the image are as follows:

- Shutter speed 1/30
- f/3.4
- ISO 125

The original image size was 3456x4608 pixels, while the final was 3156x3615 pixels. GIMP was used as the choice of image editing software, where post processing was performed to enhance the red color of the india ink, and to brighten the white background. This was achieved using the Curves feature by creating an “S” shaped curve. The image was cropped to create a symmetry in the amount of the cell that was shown, and the clone stamp tool was used to get rid of the outer boundary of the laundry detergent that could be seen in the image. The original image is displayed below.



Original Image

5 Image Thoughts

This image reveals a different form of the Hele-Shaw cell than what is traditionally displayed and studied. Initially, I had considered submitting an image of the bulbous looking finger instability, but chose to submit the version showing the reversal of the pressure gradient and enjoyed attempting to understand what was happening to create the interesting sharpened fingers. I like the asymmetry of the cell overall, since the flow only fully reached one side of the hole where the ink was introduced, and the shadows behind the ink which gives depth to the 2-dimensional flow. The challenge in this project was selecting the liquids and the lighting

setup that would best demonstrate the instability. In the future, assuring that the gap between the two plates is the same throughout the setup would help to achieve a more even radial distribution. Something as simple as installing several clamps around the edges of the two surfaces would help to address this issue in the future.

References

- [1] Faber, T.E. *Fluid Dynamics for Physicists*. Cambridge: New York, 1995. Print.
- [2] Homsy, G. M. *Viscous Fingering in Porous Media*. *Annu. Rev. Fluid Mech. Annual Review of Fluid Mechanics*. 19.1 (1987): 271-311. Web. 15 Nov. 2015.
- [3] Johns, L. E., and R. Narayanan. *Interfacial Instability*. Place of Publication Not Identified: Springer, 2013. Print.