Liquid Free Surface Splash

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"Get wet" image, made by Tianzhu Fan in September 4, 2016 at ECME Lab 165 with the assistance of Xinpeng Zhao. The purpose the image is to research the liquid splash phenomenon when a three-dimensional solid body dropped into free surface.

Second Part (Paragraph)

Worthington carried out the study on the splash when dropping a three-dimensional object into the liquid free surface in 1908. He observed different splashing behaviors when a polished sphere and a roughened sphere fall into the liquid [1]. Since then, the splash phenomenon has been studied extensively by different authors.

This phenomenon could be roughly explained as when the object arrives at the liquid surface, the liquid will fall under the pressure given by the object, and if the object's impact velocity is high enough, an air cavity will be generated at the liquid surface. To illustrate, Duez et al found that the condition to form the air cavity is dependent on a critical velocity. They build a quantitative model by theoretical method to analyze the critical velocity. What has been found is that when the object velocity is below the critical velocity, the liquid surface will remain stable. While if the impact velocity is higher than the critical velocity, an unstable liquid film will form and leads to cavity formation [2]. Because of surface tension, the liquid molecules tend to fill in the air cavity but overcompensated. If this overcompensation is limited, the travelling waves will appear and develop from the center of impingement point. While if the overcompensation is out of control, the phenomenon of splash will happen.

Since the splash is a sudden disturbance to quiescent free surface, it could be described by general free surface control equation, that is, Naiver-Stokes equations for viscous incompressible fluid. To build more precise models, some other theories for free surface has been developed such as linearized free surface wave theory. The basic equation can be depicted as follows,

$$\nabla^2 \emptyset = 0 \tag{1}$$

$$\frac{p}{\rho} = -\left(\frac{\partial \emptyset}{\partial t} + \frac{\nabla \emptyset^2}{2} + g\gamma\right) + \frac{\sigma}{\rho}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = constant\ on\ y = \gamma \tag{2}$$

$$\frac{D\gamma}{Dt} = \frac{\partial \emptyset}{\partial y} = v_y\ on\ y = \gamma \tag{3}$$

Where the free surface is elevated an amount $\gamma(x, t)$ from the static position. The $\emptyset$ is total velocity potential and σ means surface tension. R_1 and R_2 represent the principal radii of curvature. Also, J.J. Monaghan developed smoothed partible hydrodynamics method (SPH) for incompressible flows to describe free surface motion from the microscope principle [3]. In SPH equations, the interpolating points of liquid can be treat as particles. Each particle carries a mass m and velocity v. The momentum equation for particles a shows following,

$$\frac{dv_a}{dt} = -\sum_b m_b \left(\frac{P_a}{{\rho_a}^2} + \frac{P_b}{{\rho_b}^2} + \prod ab\right) \nabla_a W_{ab} + F_a \tag{4}$$

which $\prod ab$ is the shear and body viscosity, F_a means the body force and W_{ab} represents the interpolating kernel.

In the real situation, a stone which has rough and irregular surface is dropped into the water. To simplified the problem, the stone could be treated as a solid sphere. In that case, the Naiver-Stokes equations for incompressible axisymmetric two-phase flow could be applied as follows,

$$\rho\frac{\partial\vec{u}}{\partial t} + (\rho\vec{u}\cdot\nabla)\vec{u} = -\nabla p + \nabla\cdot[\mu(\nabla\vec{u} + (\nabla\vec{u})^T - \gamma\nabla\emptyset + \vec{F} \tag{5}$$

$$\nabla\vec{u} = 0 \tag{6}$$

$$\frac{\partial\emptyset}{\partial t} + (\vec{u}\cdot\nabla)\emptyset = \nabla\cdot(k\nabla\gamma) \tag{7}$$

In particular, the equation (7) is the Cahn-Hilliard equation. The $\emptyset$ means s continuous scalar variable which has constant values for different phases. For the liquid phase, $\emptyset$=1. For the gas phase, $\emptyset$=-1. Also, γ is the chemical potential and k is the mobility. Using the above equations with given boundary conditions and initial values, the liquid free surface motion can be solved by numerical method. For more research progresses on 3-dimensional objects plunging into water, Y.-M. Slocan and A.A. Korobkin give a research of energy distribution from vertical impact of a solid body onto free surface [4] and Masao Yokoyama et al state that the water splash patterns strongly depend on the surface conditions of the solid objects [5]. Because of surface tension, the water splashed form a perfect smooth surface, just like a water wall, especially shown in the right part of the photograph. Surface tension is an elastic tendency that make the surface area as less as possible. Surface tension could be described in microscopic called "surface force", which means between every arbitrary fluid element, there is an interface force in both normal and tangential directions [6].

Actually, the free surface splashing has very different kinds of applications in nature. Brodie found that raindrops splashing on soil can dispersal the seeds and microorganisms, which has positive influence on plant propagation. Also, the waterfall electrification is due to electrical charges separation under liquid splashing process [7].

Third & Fourth Part (Paragraph)

Canon DSRL T5 is applied for the image. For the experiments condition, the cup is put on the black surface and a standard lens is used about 30 millimeters away from the cup. The continuous mode is applied to catch the moment that the stone dropped into the liquid surface. The aperture is 5.6, the shutter is 1/250 and ISO setting is 1600. The lights of experiments place are bright enough thus the ray of light come from all around. The original image is in white color and a post-treatment software is used to make the whole image in blue tone. No special visualization has been used for the image.

Fifth Part (Paragraph)

Some aspects should be improved. A solid perfect sphere should be a better choice to have symmetric shape of water splashed rather than an irregular stone. While perhaps the asymmetry is also a kind of beauty. Also, a smaller object would be better because the water splashed seems somehow "out of control". Some parts of the image are not very clear thus the focus technique should be enhanced.

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[3] J. J. Monaghan. "Simulating Free Surface Flows with SPH." *Journal of Computational Physics.* 110, 399-406(1994)

[4] Y. -M. Scolan and A.A. Korobkin. "Energy distribution from vertical impact of a three-dimensional solid body onto the flat free surface of an ideal fluid." *Journal of Fluids and Structures*. 17(2003)275-286

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[7] W.C. Macklin and G. J. Metaxas. "Splashing of drops on liquid layers." *Journal of Applied Physics.* 47, 3963(1976)