Inverted Glass Containing Water Retained by Paper Card and Atmospheric Pressure

Isaac Lammers
MCEN5151: Flow Visualization
Aerospace Engineering
University of Colorado, Boulder

Introduction

The objective of the first image project was to capture a unique perspective of forces someone can encounter in their everyday lives. The image captured is intended to convey a physical force in nature and how fluid reacts when that force acts upon it. In this case the forces present in the image are gravity, atmospheric air pressure, and surface tension. It was my goal that the image should be both artistic in nature and representative of the forces without much need for explanation.

Explanation of Forces

A wine glass with a small amount of water in it was turned upside down while the paper was held up against the rim of the glass. The result is an upside down glass, where the water remains in the glass, seemingly defying gravity. However, on close inspection you can see some leaking water and a small layer of water separating the paper from the rim of the glass. Classically this experiment is done with a glass that has walls with a taper of greater than or equal to 90° (relative to the rim of the glass when pointed at the ground) which generally relies less on surface tension and more so on the pressure of the atmosphere to hold up the paper and water. I.e., It creates a better seal. I chose to use a wine glass for the aesthetic appeal. Water was chosen because it is readily available and is something people see every day; I was trying to make this image relatable to everyone. A diagram of the before set-up and in process set up can be seen below in Figure 1.

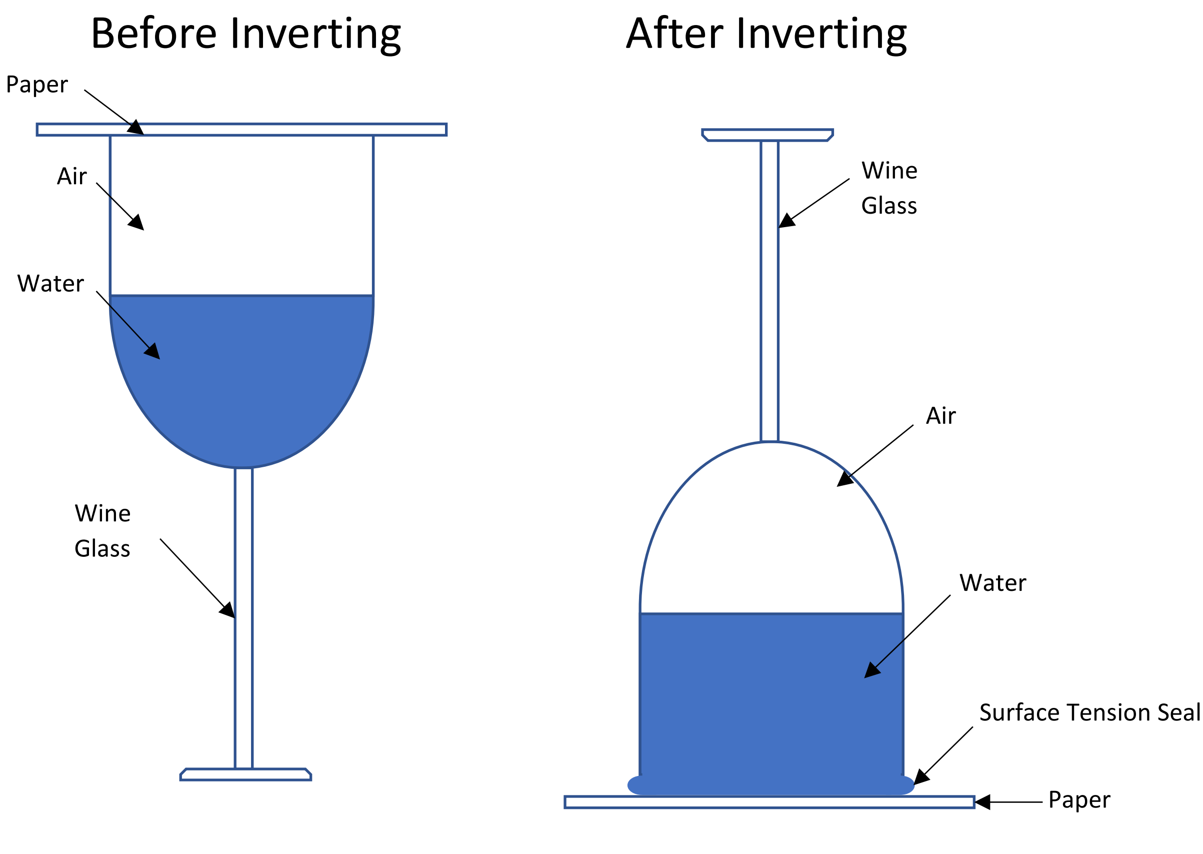

Figure 1:Experiment Setup

The inner diameter of the wine glass is 60.4 mm, the outer diameter is 63.0mm, the wall thickness is 1.3mm and the walls taper at approximately an 80° angle. The internal volume of the glass is approximately 430mL. At the time of the photo (4:56pm on 9/7/2021) the outside ambient air temperature was approximately 33°C based on historical weather data for the area, KBJC, the weather station at Rocky Mountain Regional Airport (Ogimet, 2021). Based on this I will be assuming that the water was approximately 30°C. The volume of water used was 75mL. The paper squares used were 70mm by 70mm. The value of the surface tension of water is σ = 71.20*10^-3 N/m (N.B Vargaftik, 1983). The atmospheric pressure measured that day was 30.16 hg or approximately 102.13 kPa (Ogimet, 2021).

Making the assumption that the air in the glass does not change temperature when the glass is flipped upside down it can be said that the air will be governed by Boyle's Law (Britannica, 2019). Boyle's Law states that pressure times volume of a gas will remain a constant when kept isothermal. With this we can write the following equation: P1*V1 = P2*V2, where the 1's denote the properties before the glass is flipped and the 2's denote the properties after the glass is flipped. We know the pressure of the air before the glass is flipped is simply the atmospheric pressure at P1 = 102.13 kPa, and the volume of air is simply the volume of water subtracted from the total volume of the glass which is V1 = 355mL = 3.35*10^-4 m^3. Based on an estimate that I lost 10mL of water when the glass and paper are flipped and that the paper sits about 2mm from the rim of the glass we can estimate V2 = 3.71*10^-4 m^3. With that we can simply solve for the interanal pressure using Boyle's Law. P2 = 97.66 kPa. This makes sense because we expected the internal pressure to be less than the atmospheric pressure in order for the paper to be held up and prevent the water from spilling out. By performing a quick force balance we can check to see if this is correct as well. We know that the atmosphere is applying a pressure on the paper of 102.13 kPa. We can convert this into a exerted force on the paper by using the area of the paper as the area to which the force is appiled. This force we will call F1 = 1572.17 Newtons. The force that is opposing this force is simply the weight of the paper and the weight of the water. Together these apply a downward force of F2 = 0.625 N. We can make the assumption that the internal volume of air and additional mass that it would add would be negligible but the decrease in interanl pressure does aid in the force balance some how. In addition to F2 the internal pressure of the air adds F3 to the total force applied against the pressure of the atmosphere. This force is estimated to be F3 = 343.28 N. Adding F2 and F3 together we get the total force acting against the force from the atmosphere, F4 = 343.9 N. . See the appendix for calculations of the above approximations.

We can clearly see that the atmospheric pressure is more than enough to hold up the water and the paper, but it is really the surface tension and cohesion of water that allows for this phenomena to occur. Water has relatively small molecules yet it has exceptionally high cohesion strength, meaning it wants to remain together and takes a strong force to separate it from itself (Bernard Cabane, 2005). This cohesion is what allows the paper to stick to the water and create the seal. Without the paper "adhering" to the water and creating a seal to reduce the internal pressure of the glass there is no way that the force would hold up the water and the paper. Cohesion strength comes from a combination of properties that liquid water exhibits. These are the Hydrogen Bonds that bind water molecules together, the spcific heat of the water and water's polarization (Bernard Cabane, 2005). Cabane (2005) states that the Hydrogen bond in water gets its "strength mainly from the electrosatic attraction of the non-bonding electron pairs localized on the proton acceptor molecule by the positive charges of the nuclei in the proton donor molecule." This attraction is really strong in water molecules because the distance between the molecules is incredibly small. All of this is to say that liquid water exhibits some of the strongest cohesion forces of any molecule that is a liquid at room temperature. From Varagaftik (1983) we can infer that this cohesion force decreases with an increase in temperature as the measure of the surface tension of water, σ, decreases as temperature increases in the Table on page 819. If we were to increase the temperature of the water this experiment may not even work as it could reduce the waters' ability to even "adhere" to the paper and generate the seal required to create this phenomena.

Visualization Method

To capture the image, I first set up a small studio space in my garage where I could better control the light. I used a black bed sheet hung behind the glass boxing it on three sides to reduce reflections generated by the background. The stand was placed in front of the three-sided box in the middle. I placed a wooden stool with a cutting board on top of it just below the glass to provide a reference to where the glass was in space to show the direction of gravity. I then placed a light up and slightly behind the glass, pointed at the bed sheet at a downward angle to back light the subject and reduce glares in the glass. This took some experimenting to find a balance between background reflections and glare from the light source. After some experiments with the camera settings, I settled on using an ISO 200, 1/6 sec exposure, f5.6 aperture, manual focus, a 50mm lens with an effective focal length of 75mm with my crop sensor on my Sony A6000 Mirrorless Camera body. The photo was taken on 9/7/2021 at 4:56pm. See Figure 2 for a picture of the setup.

Figure 2: Image Capturing Setup

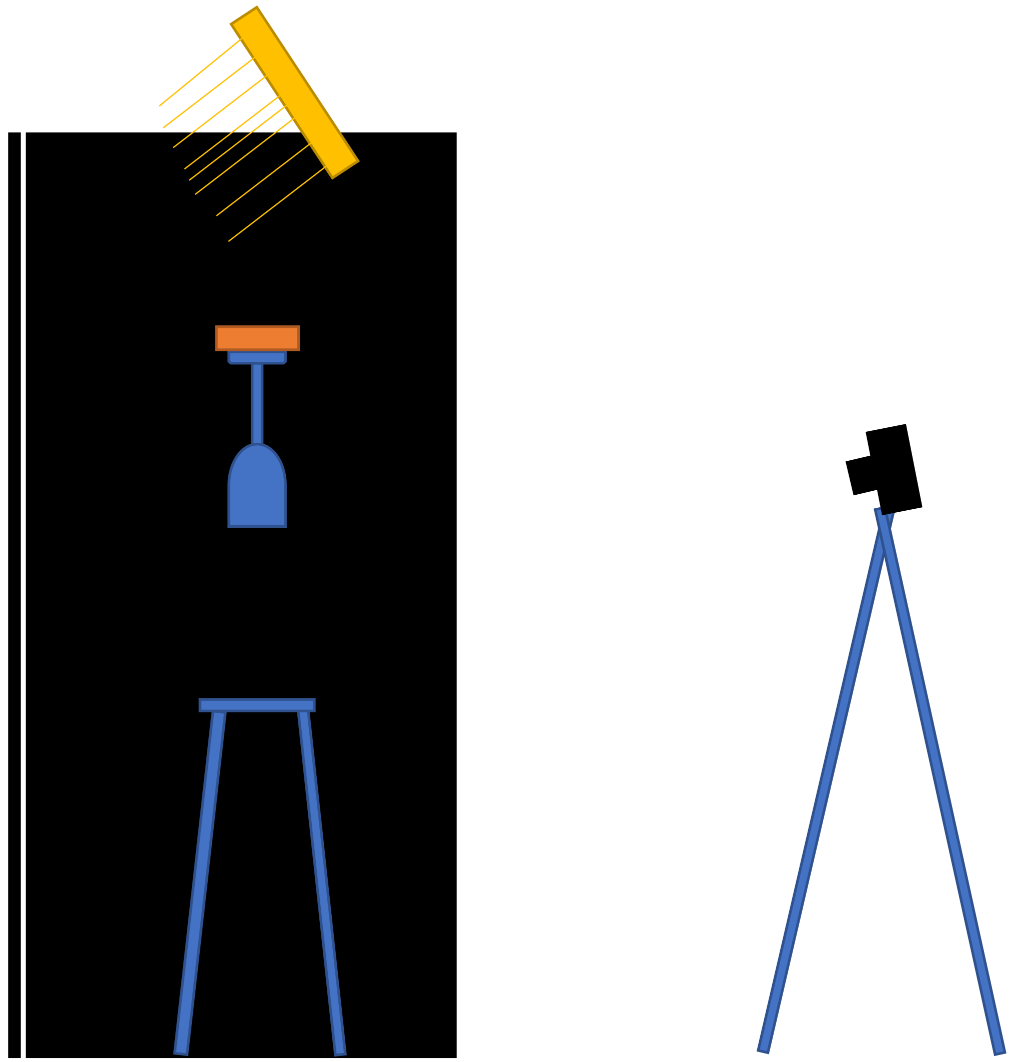

Figure 3:Lighting Diagram

The camera is absent from Figure 2 to better show the shot angle from behind the tripod. The camera was angled slightly down to reduce reflections. The water catch pan was removed before taking a photo. Figure 3 shows how the lighting was placed. The garage door was shut so no outside light could get in and the only light source was the sunlight held above the glass as shown in the figure. This allowed for greater control over the reflections in the glass.

Image Processing

I did some slight post processing to reduce saturation to give the image a black and white feel while really focusing on the highlights and subject of the photo. This really helped draw your eye directly to the subject and keeps it centered. I also added a slight vignette to reduce the viewers eye from being drawn to the lighter edges of the lightened image. I cropped the image slightly to remove the water splash on the cutting board from an earlier spill and focus in on the glass to allow it to take up more of the image space. The original and edited versions are shown below.

Figure 4:Original

Figure 5:Edited

Final Thoughts

I think that the image I captured produces a thought-provoking image that people can understand. If for a moment you stop and think you can see that there is nothing under the glass except air so it must be what is exerting the force on the paper to hold it up. It is also apparent that the water is at the relative top of the glass when turned upside down showing that the direction of gravity has not shifted in this perspective. Something that could aid in the presentation of this image is possibly adding smoke or some sort of visible gas inside the glass with the water below it to show that there is indeed air still in the glass and not some trick with a vacuum. All in all, though I think this image captures what was laid out in the objective; an image that captures everyday forces in an eye pleasing and thought inciting way.

References

Bernard Cabane, R. V. (2005). The Physics of Liquid Water. *Hal*. Retrieved from https://hal.archives-ouvertes.fr/hal-00015954/document

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Appendix

Estimate of V_2

Estimating V_2 Based on 2mm of water out of the glass and a loss of 10mL of water when cup is flipped.

Rough est. of water outside of glass

2mm x Area of the opening of the glass.

inner Dia. of Glass = 66.4 mm
outer dia of Glass = 63mm

$2mm \times \pi r^2 = 2mm \times 31.5mm^2 \times \pi$

$= 6234.49\ mm^3$

$V_{WST} = 6.234\ mL$

Water Volume = 6.234 mL
Water Leaked = 10 mL

Total Volume of that ≈ 16.25 mL

So we can add this to the V1 making the Volume when the glass is flipped (Assuming No Air Leaked in)

$V_2 = V_1 + 16.25mL = 355mL + 16.25mL$

$= 371.25mL$

$\boxed{V_2 = 3.71 \times 10^{-4}\ m^3}$ V_2

Internal Pressure Calc

$P_2 = \frac{P_1 V_1}{V_2} = \frac{102.13KPa \times 355mL}{371.25mL} = \boxed{97.66\ KPa}$ P_2

Estimate force applied by Atmosphere on the paper

$F_1 = 102.13\ KPa \cdot \frac{1000 \frac{N}{m^2}}{1 KPa} \cdot m^2$

$F_1 = 102{,}130 \frac{N}{m^2} \cdot (70mm)^2 \times \pi$

$F_1 = 102130 \frac{N}{m^2} \cdot (.070m)^2 \times \pi = \boxed{1572.17N}$ F_1

Estimate the mass of the water left above the paper after flipping the glass

Approximately 65mL of water above the paper & in the glass.

$75mL - 10mL = 65mL$
(Total start) - (leakage)

density of water @ 30°C ≈ .996 g/cm³.

$M_w = 65ml \times .996 g/cm^3$

$M_w = 65mL \times \frac{1 cm^3}{1mL} \times \frac{.996g}{cm^3} = \boxed{64.74g}$ M_w

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Force of the water & paper with the effects of gravity

Estimate the weight of the paper to be:

Density of paper ≈ 1.2 g/cm³

Volume of paper = $70mm \times 70mm \times .1778mm = 871.22mm^3 = .87122 cm^3$

Mass of paper = $1.2 g/cm^3 \times .87122 cm^3 = \boxed{1.05 grams}$ M_p

Total Mass of paper & water = 65.79 grams

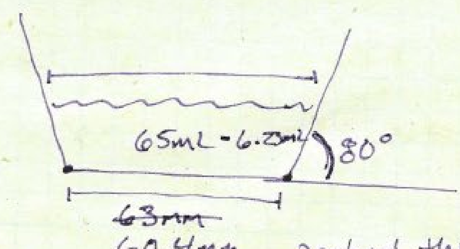
Force due to gravity = $65.79g \times \frac{1kg}{1000g} \times 9.81 m/s^2 = \boxed{.645 N}$ F_2

Estimating the Force from the internal pressure of the glass

F_3

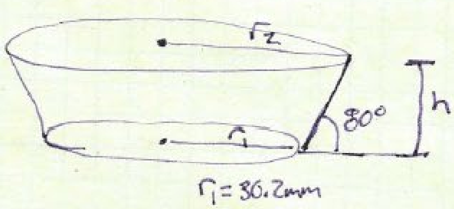
$P_2 = 97.66 kPa$ This pressure is exerted on the area of water inside the glass

Using a cone as an approximation of the glass and that Roughly 65mL - 6.234mL of water remained in the glass we can determine the surface area of the top of the water when it is flipped over.



pretend this is a section of a cone

Knowing that the Volume of our Truncated cone is 58.~~77~~mL we can estimate the internal diameter of the surface of water inside the ~~glass~~..



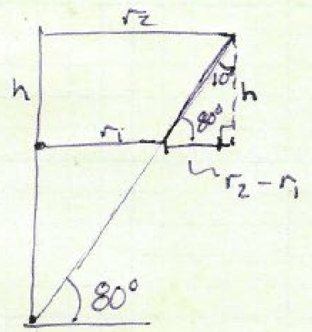
Solve for h interms of r_1 & r_2

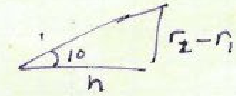
Volume of a Truncated Cone = ~~$\frac{1}{3}\pi r_2^2 h_2 - \frac{1}{3}\pi r_1^2$~~

$$= \frac{1}{3} \cdot \pi \cdot h \left(r_2^2 + r_1^2 + r_2 \cdot r_1 \right)$$

~~***~~

Solving h in terms of r_1, r_2 & cone angle





$$\tan(10) = \frac{r_2 - r_1}{h}$$

$$h = \frac{r_2 - r_1}{\tan(10)}$$

$$V = \frac{1}{3}\pi\left(\frac{r_2 - r_1}{\tan(60)}\right)\left(r_2^2 + r_1^2 + r_2 \cdot r_1\right)$$

~~V=6~~

$$V = 58770\,mm^3 = \frac{1}{3}\pi\left(\frac{r_2 - 30.2mm}{\tan(10)}\right)\left(r_2^2 + 30.2mm^2 + r_2 \cdot 30.2mm\right)$$

Using a solver (wolfram alpha)

$$r_2 \approx 33.45mm$$

Knowing this the internal pressure force acting on the surface of the water is.

$$F_3 = 97.66\,kPa \times \frac{\frac{1000N}{m^2}}{1\,kPa} \cdot (.03345m)^2 \times \pi = 343.28N$$

Total Force Exerted against the force of the atmosphere

$$F_4 = F_2 + F_3 = 343.28N + 0.645N = \boxed{343.9N}$$ F_4