Jacob Haimes
Assignment 2 Report
ARTF 5200
Assisted by Leah Selman

Propelled Paint

10/11/2021

Overview

This picture was taken with the intent to capture moderately forced fluid thread breakup. Specifically, the fluid experiencing thread break up was a paint-water mixture, and it was dropped into a stream of forced air. Leah Selman assisted in the set up of the photograph, and poured the paint as the picture was taken.

The Flow

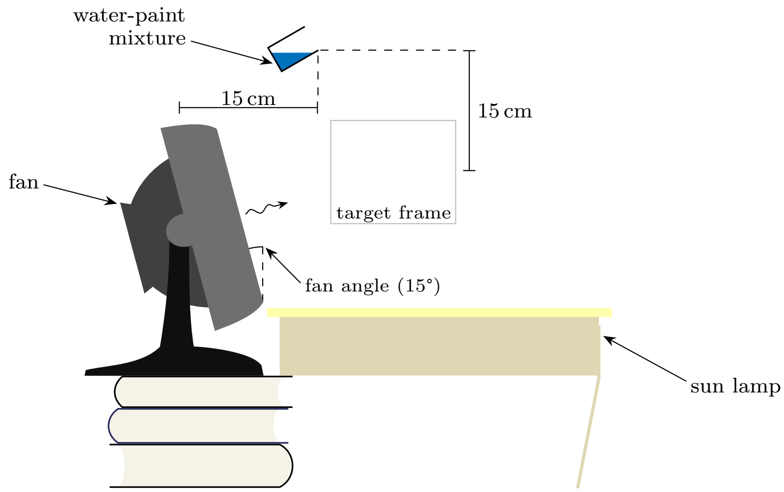

Figure 1: Diagram of the setup used for the picture. All values in this diagram are approximate.

First, a *thread-fluid* with desirable properties was concocted. Crayola paint was deposited into a small beaker, which was then thinned by recursively adding a small amount of water and thoroughly mixing the fluids. In between each iteration, the fluid was examined to determine if it had reached the target viscosity. For this photograph, I wanted a *thread-fluid* with viscosity similar to that of olive oil. The mixture was then slowly poured into a fast flowing ($0.13\,\mathrm{m^3/s}$) stream of air [1], which was at an angle approximately 15° with the horizontal. This air stream was was created by a small Vornado 5303 Small Air Circulator on the highest setting. A diagram of the set up, including the location of the target frame, can be seen in Fig. (1).

In order to characterize the situation correctly, we will first determine the Reynolds number and the Weber number of a single droplet. Before delving into the numbers, we recall the physical meaning of the two dimensionless parameters. The Reynolds number, defined in Eq. (1), is the ratio of inertial to viscous forces within a fluid, and the Weber number, defined in Eq. (2), is the ratio of inertial forces of the medium fluid to the surface tension of a jet or droplet. In the aforementioned equations, each symbol represents different fluid properties of the *thread-fluid* (ρ is density, u is velocity, L is the characteristic length, μ is the kinematic viscosity, and σ is the surface tension).

$$\mathrm{Re} = \frac{\rho u L}{\mu} \tag{1}$$

$$\mathrm{We} = \frac{\rho_g u^2 L}{\sigma} \tag{2}$$

For this analysis, we will assume that a single droplet of our *thread-fluid* is perfectly spherical as it enters the air stream (meaning that our characteristic length L will be the diameter), and has fluid properties similar to olive oil. The most relevant properties of olive oil can be found in Table 1.

Property	**Value**	**Units**
density	908.7	$\mathrm{kg/m^3}$
kinematic viscosity	74.1	$\mathrm{mPa\,s}$
surface tension	31.9	$\mathrm{mN\,m}$

Table 1: Some fluid properties of olive oil, which will be used for our analysis. All values in this table come from "Density, viscosity, and surface tension of five vegetable oils at elevated temperatures: Measurement and modeling," from the *International Journal of Food Properties* [2].

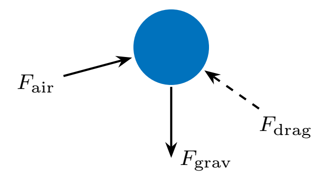

Figure 2: Free body diagram of a single droplet of paint as it enters the target frame. Note that while F_{grav} and F_{air} have (relatively) constant direction over our target frame, F_{drag} will not, as it drag force always opposes the direction of movement with a magnitude proportional to velocity squared.

Upon reexamination of Eq. (1) and Eq. (2), we see that we need to determine velocity of the fluid, and it's characteristic length. To understand the velocity as a function of time, we draw a free body diagram to study the external forces, which is seen in Fig. (2).

Because of the complex nature of this problem, we will isolate analysis to immediately before, and immediately after the droplet enters the air stream. With this knowledge, qualitative assertions will be made about the remainder of a droplets trajectory. If we assume that a droplet enters the air stream 7.5 cm below the drop height, it's velocity can be calculated.

$$\begin{aligned} & \Delta x = \tfrac{1}{2}at^2 \qquad (3)\\ \implies\ & 7.5\,\mathrm{cm} = \tfrac{1}{2}\left(9.81\,\mathrm{m/s^2}\right)t_{\mathrm{entry}}^2 \\ \implies\ & t_{\mathrm{entry}} = \sqrt{\frac{2(0.075\,\mathrm{m})}{9.81\,\mathrm{m/s^2}}} \approx 0.124\,\mathrm{s} \\ & \begin{cases} \Delta v = at \\ v_0 = 0 \end{cases} \qquad (4)\\ \implies\ & v_{\mathrm{entry}} = \left(9.81\,\mathrm{m/s^2}\right)t_{\mathrm{entry}} \approx 1.21\,\mathrm{m/s} \end{aligned}$$

The instant prior to the droplet hitting the air stream can be characterized by the Reynolds number of the air passing around the droplet, and the Weber number at that time, calculated in Eq. (5) and Eq. (6) respectively. We will use a density of $\rho_g = 1.204\,\mathrm{kg/m^3}$ for air [3].

$$\mathrm{Re}_{g,\,\mathrm{ambient}} = \frac{\left(1.204\,\mathrm{kg/m^3}\right)\left(1.21\,\mathrm{m/s}\right)\left(5\,\mathrm{mm}\right)}{1.825\times10^{-5}\,\mathrm{Pa\,s}} \approx 397.8 \qquad (5)$$

$$\mathrm{We}_{\mathrm{ambient}} = \frac{\left(1.204\,\mathrm{kg/m^3}\right)\left(1.21\,\mathrm{m/s}\right)^2\left(5\,\mathrm{mm}\right)}{31.9\,\mathrm{mN\,m}} \approx .275 \qquad (6)$$

These values make sense, as the stagnant air should not have a high Reynolds number, and no fluid breakup is occurring [4]. After the droplet enters the air stream, there are a few key differences we must account for. The first is that the velocity of the air relative to the droplet will increase and change direction. Before calculating this relative velocity, we must first obtain the velocity of the air stream. Using specifications from a retailer of the Vornado 5303 fan [1], Eq. (7), Eq. (8), and previously calculated values, we determine the relative speed of the air around the droplet. In these equations, our new variables are volumetric flow rate of the air exiting the fan ($\dot{V}$), the cross sectional area of the fan (A_{fan}), the velocity of the air exiting the fan (u_{stream}), and the relative velocity of the air exiting the fan with respect to the droplet (u_{rel}).

$$u_{\text{stream}} = \frac{\dot{V}}{A_{\text{fan}}} \tag{7}$$

$$\implies \quad u_{\text{stream}} = \frac{0.13\,\mathrm{m^3/s}}{\frac{\pi}{4}\big(0.285\,\mathrm{m}\big)^2} \approx 2.038\,\mathrm{m/s}$$

$$u_{\text{rel}} = ||\vec{u}_{\text{entry}} + \vec{u}_{\text{stream}}|| \tag{8}$$

$$\implies \quad u_{\text{rel}} = \Big|\Big|1.21\big(0\hat{\imath} - \hat{\jmath}\big) + 2.038\big(\cos(15^\circ)\hat{\imath} + \sin(15^\circ)\hat{\jmath}\big)\Big|\Big|$$

$$\implies \quad u_{\text{rel}} = ||1.968\hat{\imath} - 0.683\hat{\jmath})|| \approx 2.083\,\mathrm{m/s}$$

This change in relative velocity of the air also contributes to changes in the Reynolds number and the Weber number. The new Reynolds and Weber numbers are calculated in Eq. (9) and Eq. (10), respectively.

$$\mathrm{Re}_{g,\text{stream}} = \frac{\big(1.204\,\mathrm{kg/m^3}\big)\big(2.083\,\mathrm{m/s}\big)\big(5\,\mathrm{mm}\big)}{1.825 \times 10^{-5}\,\mathrm{Pa\,s}} \approx 672.2 \tag{9}$$

$$\mathrm{We}_{\text{stream}} = \frac{\big(1.204\,\mathrm{kg/m^3}\big)\big(2.083\,\mathrm{m/s}\big)^2\big(5\,\mathrm{mm}\big)}{31.9\,\mathrm{mN\,m}} \approx .784 \tag{10}$$

We can see that upon entering the stream, our Reynolds number and Weber number increase instantaneously, which makes sense in the context of what the parameters mean physically, as well as previous research on them [4]. As the droplet continues forward, both the Reynolds number and the Weber number will continue to increase as the droplet picks up speed, until fluid break up occurs. When this happens, the changes in geometry of the component droplets will result in a drastic reduction to both dimensionless parameters (relative to the larger droplets).

Visualization Techniques

This image utilizes a combination of Crayola washable paint and water as the *thread-fluid*, making the fluid more visible and vibrantly colored. Prior to the shoot, a sun lamp was balanced on top of some books, resulting in a relatively horizontal plane emitting light in the upwards direction. This set up was chosen in order to light the droplets from below, and accentuate the contrast between them and the white background. A diagram of the set up can be found in Fig. (3).

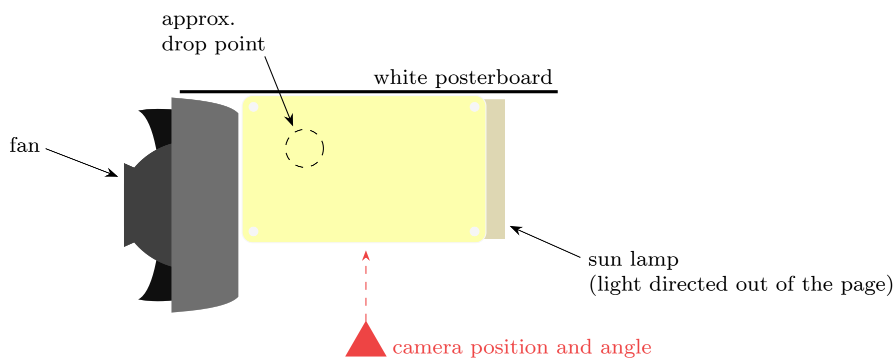

Figure 3: Birds eye view diagram of light setup including approximate location of camera lens during shooting. Diagram is not necessarily to scale.

Photographic Techniques

The pictures were taken with a Nikon D90 using a AF-S DX VR Zoom-Nikkor lens. For the photos of the pink *thread-fluid*, an aperture of f/5.3, an exposure of 1/2000, and a focal length of 75 mm were used, while the photos of the blue *thread-fluid* used an aperture of f/6.3, an exposure of 1/4000, and a focal length of 48 mm. The final image was actually a composite of 6 different pictures. For 5 of these pictures, the background was removed in GIMP via masking, and then each layer was imported onto the 6^{th} frame, with their positions adjusted slightly in order to prevent any droplets from being obscured. Prior to this overlaying process, each image underwent post processing in darktable, which primarily utilized lens correction, adjusting white balance, bringing in the edges on the base curve, heavily modifying color zones, and increasing local contrast. The original images, in lower resolution, can be seen in Fig (4) for context regarding initial image quality and field of view.

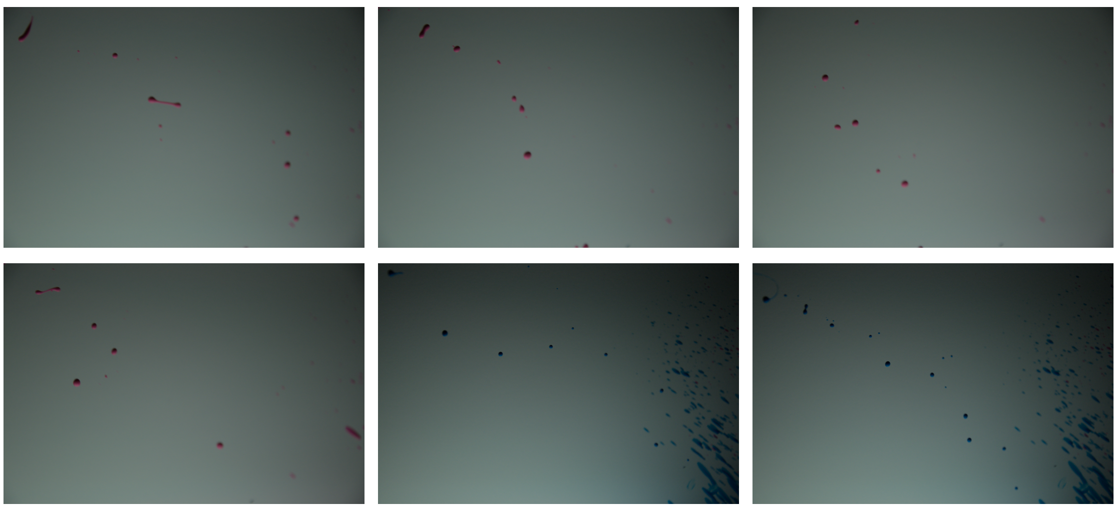

Figure 4: This was the original photo taken, unedited other than to reduce it's size for display purposes. The original images were 4310×2868 pixels. The picture on the bottom right was the base photograph for the final image.

Final Thoughts

This image provides a very interesting insight into a field I did not yet have much experience with: fluid breakup. It visualizes how a droplet behaves as its Weber number changes because the droplets on the right side of the frame should have higher Weber numbers than similarly sized droplets of the left side of the frame. I am overall relatively pleased with my final product, particularly in how I was able to capture a high contrast with the colors I chose, as well as underlighting and post processing. I would have preferred if I could get the droplets more in focus in general, but I also understand that capturing falling droplets in focus is very difficult. One idea for a future photograph that also explores fluid breakup would be to pour a slightly more viscous fluid into an air stream that was not moving as fast. This would allow for the visualization of the Plateau-Rayleigh instability as the thread is subjected to forces in addition to gravity.

References

[1] Ubuy, “Buy Vornado Small Whole Room Air Circulator Fan,” Best Online Shopping Store for Electronics, Fashion, Home Improvement & More in Maldives, 2021. [Online]. Available: https://www.ubuy.vn/en/product/1RQ5KI8-vornado-5303-small-whole-room-air-circulator-fan-with-base-mounted-controls-3-speed-settings-multi-d. [Accessed: 05-Oct-2021].

[2] S. N. Sahasrabudhe, V. Rodriguez-Martinez, M. O’Meara, and B. E. Farkas, “Density, viscosity, and surface tension of five vegetable oils at elevated temperatures: Measurement and Modeling,” International Journal of Food Properties, pp. 1–17, 2017.

[3] Engineers Edge LLC., “Kinematic viscosity table chart of liquids - engineers edge,” Engineers Edge - Engineering, Design and Manufacturing Solutions, 2021. [Online]. Available: https://www.engineersedge.com/fluid_flow/kinematic-viscosity-table.htm [Accessed: 06-Oct-2021].

[4] T. Kékesi, G. Amberg, and L. Prahl Wittberg, “Drop deformation and breakup,” International Journal of Multiphase Flow, vol. 66, pp. 1–10, 2014.

NOTE: academic literature used are citation numbers 2 and 4.