IV 4 Report
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Introduction

This report outlines my experience creating a video of the Tea Leaves Paradox for the third group project of the semester. This project was inspired by the youtuber Steve Mould, and his video explaining the Tea Leaves/Inverted Vortex Paradox (Found here). The goal of this project was to replicate the visualization created by Steve Mould, and to use that to better understand the physics of the paradox.

Background

The Tea Leaves Paradox got its name from Albert Einstein, who first noticed the phenomenon when he stirred a cup of tea. He was puzzled by the fact that the solid tea leaves were gathering in the center of the cup rather than being pushed to the edges due to rotating forces. Let's investigate…

The stirring motion causes the surface of the fluid to form a parabolic shape.This is caused by a velocity gradient that changes with radius.

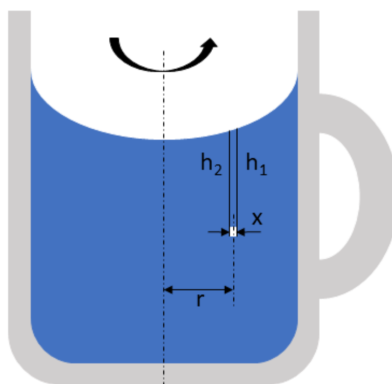

Figure 1: Diagram of a cup of tea being stirred.
Provided by medium.com

$$F_{rotation} = mass * angular\ velocity^2 * radius \quad (1)$$

With this equation above, it is clear that if the angular velocity is lower, the centrifugal force felt by the water/tea will be lower accordingly. Interestingly, the angular velocity is constant throughout the cup, with the overall velocity changing with radius.

$$Velocity = angular\ velocity * radius \quad (2)$$

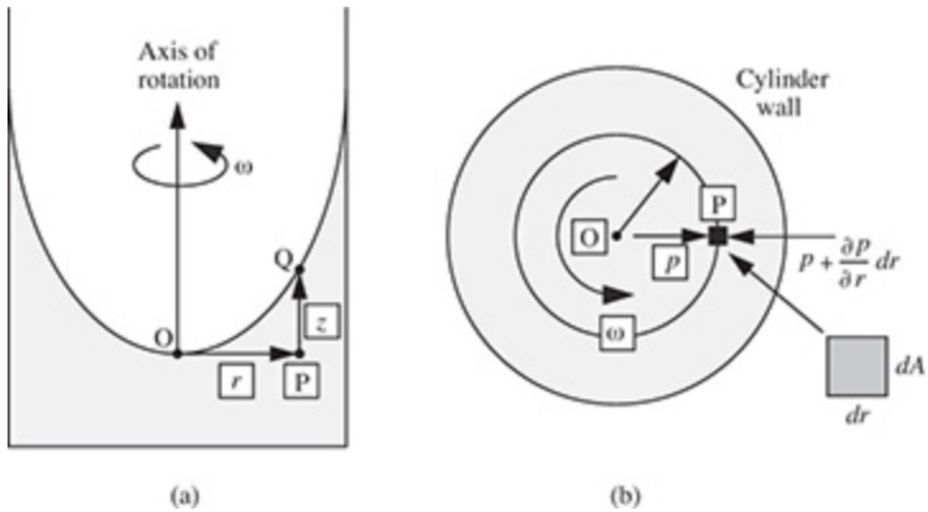

Figure 2: Pressure changes for rotating cylinder: (a) elevation (b) plan.
Provided by https://www.informit.com/articles/article.aspx?p=2832417&seqNum=7

Analyzing the infinitesimally small area above in figure 2…

$$m_{dAdr} = \rho * dA * dr \quad (3.1)$$

$$F_{net\ pressure} = ((p + \frac{\partial p}{\partial r}dr) - p)dA \quad (3.2)$$

Now, we can combine equations 1, 3.1, and 3.2 to get an equation for the partial differential of pressure with respect to radius.

$$((p + \frac{\partial p}{\partial r}dr) - p)dA = \rho(dA)(dr)(\omega^2)(r) \quad (3.3)$$

Simplifying

$$\frac{\partial p}{\partial r} = \rho\omega^2 r \quad (3.4)$$

$$\int \partial p = \int_0^r \rho\omega^2 r\, dr \quad (3.5)$$

An assumption can be made that integrating ∂p will give us the known equation that pressure changes with respect to height and density[1].

$$\int \partial p = \rho g z* \quad (3.6)$$

We are then finally left with an equation that explains why there is a parabolic surface shape of the fluid.

$$z = \frac{\omega^2 r^2}{2g} \quad (3.7)$$

Now armed with the knowledge behind the behavior of the surface shape of the fluid, we can make some informed assumptions about why tea particles would pool in the center of the spinning fluid.

Since we know that the pressure changes with the head height of the fluid, there is clearly a difference in pressure between the center of rotation and the out edge of the cup. The parabolic shape caused by stirring creates a larger head height at the edge of the cup, thus the pressure is greater.

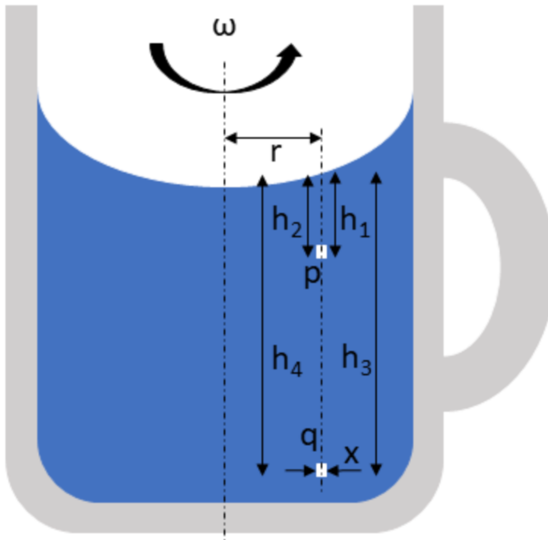

Figure 3: Diagram with two infinitesimally small areas in the same radial position.
Provided by [medium.com]

For this scenario, pretend the fluid has just stopped being stirred constantly.In this diagram, points p and q are experiencing the same difference in pressure on their inner and outer edges. This means that the net force acting on each element is equal. There is one final bit of physical trickery at play. The fluid that is in contact with the surface of the cup has zero velocity due to the no slip condition. The speed of the fluid slowly increases as it moves further away from the cup's surface. In the scenario in figure 3, the point q is closer to the cup surface, and thus has a slower angular velocity and acceleration. What this means is that at point q, the net force in the direction of the axis of rotation is greater than at point p[2]. So, once the fluid is no longer being stirred, particles will experience different magnitudes of deceleration towards the center of rotation dependent upon their distance relative to the cup's surface. This creates a vortex pattern depicted in the figure below.

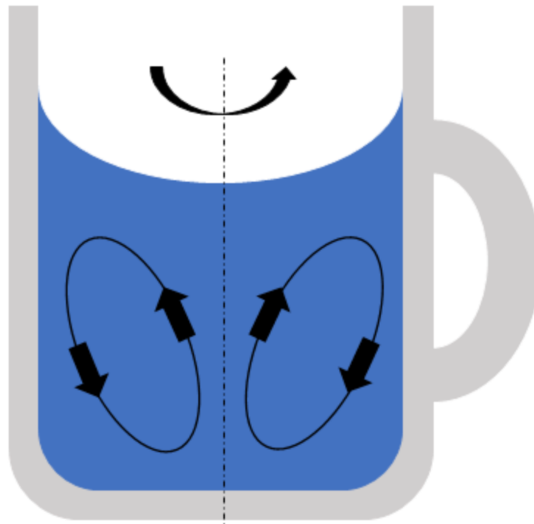

Figure 5: Vortex pattern that causes the behavior of the Tea Leaves Paradox
Provided by [medium.com]

Once tea leaves, or fluids of greater density, are introduced into the cup, they are propelled into the center of the cup, giving us our Tea Leaves Paradox. The higher density fluid will create a short column in the center, in which the height is determined by both the speed of the stirring and the difference in densities between the fluids.

Photography

To recreate the video by Steve Mould, our team used canola oil and dyed water as the two different density fluids. The two fluids were mixed in a large glass beaker and stirred using a small stir rod. Anders Hamburgen was kind enough to lend me his camera for the shoot, so I stirred the fluids while he recorded the video. Rather than use the flash on the camera, an external LED attached to the camera was used to light the subject.

The camera was placed on the same counter surface as the glass beaker, and was placed ~1 ft away from the beaker. The camera used was a Sony a6300 digital camera with a 16-50 mm lens that was zoomed in to the focal length of 50mm. With a focal length of 50mm, the diagonal FOV angle is 46.8 degrees. The video was captured at 30 fps with a resolution of 1920 x 1080p. The ISO was set to 1200 and the aperture was set to f/5.6. These settings were optimal for focusing on the fluid inside the beaker and capturing the full frame with the beaker.

Conclusion

While I am happy with the final video, I would like to repeat this experiment using mineral spirits instead of canola oil. This is because the densities of oil and water are extremely similar (oil has a density of 915 kg/m^3 and water has a density of 997 kg/m^3). This caused bubbles of water in the oil to remain in the mixture. Mineral spirits have a density of 790 kg/m^3 , and provide a much more distinct visualization of the Tea Leaves Paradox.

References

[1] J. Wilkes, “Fluid Mechanics for Chemical Engineers” 3rd Edition (2017) https://www.informit.com/articles/article.aspx?p=2832417&seqNum=7

[2] S. Joshi, “Why Do Tea Leaves Gather At The Center Of A Cup After Stirring?” (2020) http://physics-depristine.blogspot.com/2011/12/einsteins-tea-leaves-paradox-and-its.html