WORTHINGTON JET FORMATION AND PINCH-OFF

GET WET, FALL 2023

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INTRODUCTION

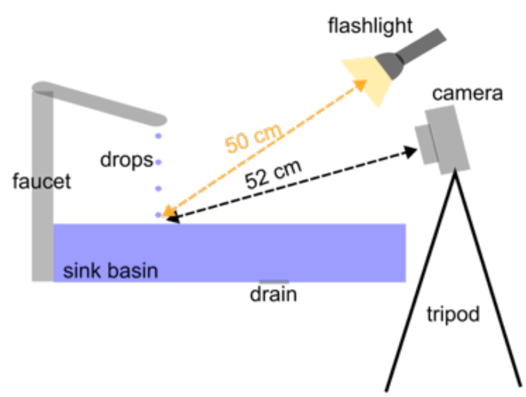
I captured this image of a Worthington after observing that the sink faucet in my Airbnb[1] was leaking intermittently. As a part of the Fall 2023 Get Wet assignment for the Flow Visualization class at the University of Colorado Boulder, I photographed the impact of a droplet with the surface of a liquid pool. This image depicts the rebounding jet (i.e.,"Worthington" jet) that arises after a droplet of fluid impacts the surface of a pool of liquid. This image captures the moment the upper portion of the jet begins to pinch off.

SETUP

To induce the collision between a falling droplet and a pool of water, I filled a sink basin with water to a level about halfway between the basin floor and the upper edge. By filling the basin only halfway, I hoped that the back wall of the silver sink would reflect light from the light source (i.e., a flashlight) to help illuminate the droplet impact. A sink stopper, inserted into the drain, limited flow out of the sink during this experiment. To avoid capturing the drain in my image, I turned the faucet head about 50 degrees clockwise to hover over a less distracting portion of the sink basin.

Choosing the ideal flow rate to output droplets took some trial and error. If I opened the faucet too much, the faucet emitted a full stream of water (not droplets), or the droplets fell from the faucet at a rate faster than I could depress my shutter button. If I opened the faucet valve too little, I had to wait a substantial period between shutter depressions, which slowed how fast I could capture and review my images. I settled on a valve position that emitted a single droplet about once every two and a half seconds (2.5 s). This period between droplets allowed the surface waves from the previous droplet-surface collision to dissipate before the next droplet impacted the surface.

To light my setup, I lit the field with a 600 lumen LED flashlight. I positioned my camera on a tripod just above the edge of the sink basin. In this setup, my tripod rested slightly above the edge of the counter with the lens of my camera about 52 cm away from the drop impact in the back corner of the sink. I held the flashlight just above, to the left, and 2 cm in front of the camera lens – about 50 cm from the droplet impact. I aimed the LED flashlight to a point just behind the droplet impact so that some of the light reflected off the silver wall and some of the light directly illuminated the jet.



[1] I would like to thank my AirBnb host, Blayne Brown, for providing the facilities used to capture this image.

PHYSICS : DROPLET REBOUND, JETS, AND PINCH-OFF

In my image, a Worthington jet rises from the surface surrounded by radiating waves. Worthington jets appear as a result of the impact of a droplet of fluid with a fluid surface. However, not all drops that collide with a pool generate Worthington jets. According to Hsiao et al., drops can coalesce with the pool of fluid, rebound off of the surface of the pool and float, or form a jet or splash ("Drop Impact," 2023; Hsiao et al., 1988). The ratio of inertial forces of the falling droplet to the surface tension forces of the pool of fluid are what decide whether the droplet bounces, floats, splashes, or forms a jet (Hsiao et al., 1988). Outside of this experiment, we can see Worthington jets we poor milk into our coffee, when raindrops collide with a body of water or wetted surface, and even in the transmission of pathogens (Cai et al., 2022; van Rijn et al., 2021)

Worthington jets were first observed and described by Arthur Mason Worthington in 1876 (Worthington & Clifton, 1876). Worthington reported on the behavior of water drops and mercury drops impacting a pane of smoked glass. Both fluids left an imprint on the smoked glass resembling radiating circles, which Worthington called "symmetrical and beautiful." He also noted that some droplet impacts created arms "…considered as free cylinders of liquid, …[that are] in equilibrium till the length bears a certain proportion to their diameter, after which they will tend to split each into a row of drops." Here Worthington was describing the formation and pinch-off of jets.

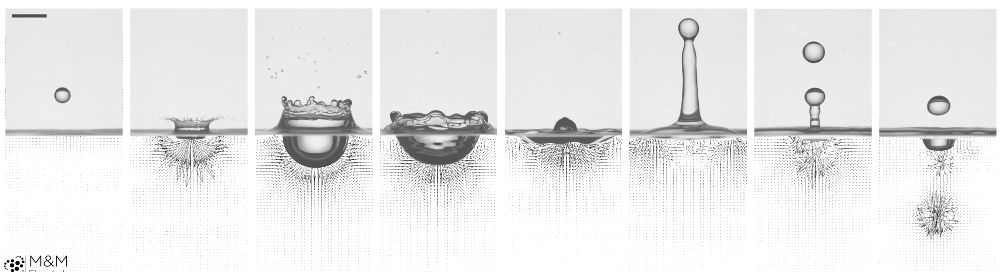


Figure 1: Images of a drop colliding with a liquid surface, forming a subsurface bubble, and creating a jet that pinches off (Sharma et al., 2022)

When a droplet impacts a liquid surface with sufficient inertial force, the impact overcomes the surface tension force that tends to maintain the stability of the surface when not disturbed. Furthermore, sufficient inertial force from the collision may overcome the gravitational pull on that liquid surface that tends to maintain the surface in a planar formation perpendicular to the direction of gravity. All of this means that with enough inertia (a combination of acceleration and mass), a droplet can disturb the surface of the impacted liquid (Hsiao et al., 1988). Once the droplet has broken through the surface of the pool, an envelope of air forms around the droplet, as shown in Figure 1. Eventually, as the droplet sinks, that envelope pinches off, forming a bubble. As that bubble rises to the surface, it forces fluid upward rapidly displacing fluid above the bubble (Hsiao et al., 1988; Sharma et al., 2021). This motion forms a jet as shown in Figure 1. (Cai et al., 2022; Gekle & Gordillo, 2010). The jet extends upward until the surface tension forces and gravitational forces on the jet start to decelerate the jet's rise. At that point, a wave within the jet body can initiate the formation of a smaller diameter neck as we see in my image (Shinjo & Umemura, 2010). When that neck is small enough, the edges of the jet converge to pinch-off the upper portion of the jet from the lower portion.

Surface tension plays a critical role in the formation of Worthington jets. In fact, if one mixes a surfactant like dish soap into the pool of fluid, that surfactant reduces the surface tension of the pool's surface enough to prevent the formation of Worthington jets (Cai et al., 2022). The role of surface tension is evident when we look at the Weber number (We), which is the ratio of the inertial force of the disruption (here the falling droplet's impact with the pool of fluid) over the surface tension or "capillary" force of the pool of fluid (Sen et al., 2023). The Weber number gives us:

$$We = \frac{\rho V^2}{\sigma L} = \frac{\text{inertial force}}{\text{surface tension force}}$$

where ρ is the density of the fluid (0.9970 g/mL for water at 25°C), V is the velocity of drop (value estimated below), L is diameter of drop (~0.5 cm)[2], and σ is the surface tension of the impacted surface (72.0 mN/m). To estimate the velocity of the droplet, I took high speed video at 120 frames per second of seven droplets falling. Over these seven videos, I observed the droplets within the view of the camera frame on average over 4 frames. In other words, on average, each droplet fell across the frame before hitting the water for 4 frames. I then measured how much distance above the water's surface lies within the frame with a ruler. Since the frame represents 10 cm vertically above the water surface, I estimated that the droplet traverses that height over the 4 frames, yielding an average velocity of 10.0 cm/(4 frames * 1/120 s/frame) =300 cm/sec = 3.0 m/s[3]. From this, we can calculate the Weber Number as:

$$We = \frac{0.9979\ g/mL * (3.0\ m/s)^2}{72\ mN/m * 0.5\ cm} = 25$$

Here we see that the inertial force far exceeds the surface tension force of the water, which means we will likely see some disruption on the surface. Hsiao et al. suggests that jets form in systems with Weber numbers greater than ~8 (Hsiao et al., 1988). With a Weber number of approximately 25 in the captured experiment, we can expect to see a jet.

In the featured image, we also see the formation of concentric vortex rings surrounding the jet. From Hsiao et al., we know that the Froude number predicts whether vortex rings form (Hsiao et al., 1988). The Froude number is another dimensionless number that describes the ratio of inertial and gravitational forces. The Froude number tells us if the inertial force of the impact is enough to overcome the tendency for gravity g to maintain the fluid surface in a plane orthogonal to the direction of gravity. The Froude number is defined as:

$$Fr = \frac{V}{\sqrt{gL}} = \frac{\text{inertial force}}{\text{gravitational force}} = \frac{3.0\ m/s}{\sqrt{9.8\ m/s\text{^}2 * 0.5\ cm}} = 14$$

In the featured image, the Froude number indicates that our inertial force from the impact of the droplet exceeds the gravitational force. Hsiao et al. provide a useful chart for predicting what sorts of Froude and Weber numbers we need to see a jet and vortex generation as shown in the Appendix. In our case, both our Weber number and Froude number exceed the thresholds described by Hsiao et al., which supports our observation that a jet and ripples emerge from the impact of a droplet of the size and velocity depicted in the image.

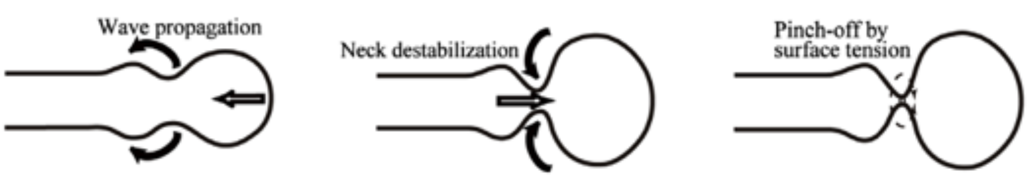


Figure 2: An illustration of jet pinch-off (Shinjo & Umemura, 2010)

[2] I estimated the size of the droplets by holding a ruler just behind the flow and captured a set of images that show the droplet in front of the ruler. This may be a slight overestimate of the droplet size.
[3] The terminal velocity of a raindrop is about 9 m/s (Gunn & Kinzer, 1949). In my experiment, the droplets falling from the faucet will not have had sufficient time to reach terminal velocity. This is to say that my velocity estimate might be an underestimate, but I would still not expect the droplet velocity to be anywhere close to the terminal velocity over the fall distance in this experiment.

In the experimental setup pictured in the feature image, the jet rises from the surface over an average of 11 frames to a peak height around 4.5 cm above the water's surface[4]. This means, the jet's velocity is around 50 cm/s or 0.5 m/s, substantially slower than the falling droplet. As the jet decelerates, a wave propagates down the length of the jet causing the neck of the jet to destabilize as shown in Figure 3 (Shinjo & Umemura, 2010). Eventually, surface tension causes the neck to collapse, pinching off a bulb or droplet like we see in the captured image (Shinjo & Umemura, 2010).

FLOW VISUALIZATION

This image relies on the different indices of refraction between the air surrounding the jet and the water in the pool. Water has a higher index of refraction than air (i.e., 1.33 compared to 1.00), which means the light slows down when it enters the water in the image ("Refractive Index," 2023). That refraction allows us to see the boundary between the water and the air clearly. In the featured image, I bounced lighted off the back of the sink wall to illuminate the flow. I was fortunate to have access to a metallic sink that reflected light nicely. The reflectiveness of the back wall of the sink allowed me to experiment with lighting placement and ensure proper illumination of the jet and ripples.

PHOTOGRAPHIC TECHNIQUES

To generate this photograph, I used an Olympus E-MD10 Mark II Camera using the Manual mode with focal length of 42 mm, an aperture of f/5.6, and exposure time of 1/1000, ISO 500, and manual focus as outlined in Table 1. In this image, the camera lens lies about 52 cm from the location of drop impact in the sink while the camera body abuts the edge of the sink on a tripod. The boundary of the sink restricted the location of the camera body, requiring some zoom to ensure the drop filled most of the captured frame.

Table 1: Camera and Image Settings

Camera	Olympus E-MD10 Mark II
Lens	M. Zuiko Digital 14-42mm 1:3, 5-5.6
Light Source	600 Lumen LED Flashlight
Focal Length	42 mm
ISO Speed Rating	500
Aperture	f/5.6
Shutter Speed	1/1000
Camera Modes	M shooting mode, Manual focus (MF), Silent Sequential shooting
Frames per second captured	11 fps in silent sequential shooting mode
Raw Image Size	4608 × 3456
Edited Image Size	4444 × 2696
Edited Image Resolution	72 pixels/inch

Since drops fall from a dripping faucet rapidly (3+ m/s) and the waves generated, I experimented with exposure times to ensure I captured the phenomena. Ultimately, the waves and jets appeared most clear with a faster shutter speed (1/1000). If the jet was moving less than 50 cm/s at the time of exposure, with a shutter speed of 1/1000, the materials in the frame should move only about 0.5 mm during the exposure. This shutter speed seems sufficient to prevent the blurry artifacts we see when an object moves too much during the exposure period. Without changing the field of view, my images were incredibly underexposed. Fortunately, a 600 lumen LED flashlight purchased from Sam's Club was adequate to illuminate the field sufficiently given the field of view was limited by the high shutter speed. I opted for a mid-range ISO (sensitivity to light) value of 500, which yielded a balance between

[4] This velocity estimate was calculated in the same manner as the droplet falling velocity: averaged over seven videos taken at 120 fps. The height of the jet was approximated based on the known frame height (10cm above the water surface).

Original Image **Edited Image**

capturing the drop impact effectively and not becoming too grainy. In retrospect, my ISO could have been a bit higher to make the RAW image a bit brighter.

I also employed the multiple exposure feature of my camera to ensure I captured the full range of impacts with the water surface. Because I was unable to depress the shutter fast enough to capture the precise moment of the jet formation, I used the multiple exposure settings built into my camera. For a single shutter depression, my camera captured 11 frames over a second in the Silent Sequential Shooting mode. This allowed me to capture a sequence of frames depicting the drop falling, colliding with the surface, jets forming, and pinch-off. From this sequence, I selected my final image. I opted for a silent shutter in this case to limit the vibration that naturally occurs when the shutter closes. This helped prevent some of the blur that might have occurred if the shutter inadvertently shook the camera during the exposures.

Figure 3: Light levels of the Original (left) and edited (right) image, depicted as histograms.

Post-Processing

In post-processing, I used darktable to crop the image and remove the back wall of the sink. Droplets had adhered to the sink wall, creating a lumpy and distracting backdrop for my image. Since the original image was effectively already monochrome with a few blue highlights, I opted to embrace this feature and make my image fully black and white. I think this decision augments some of the natural highlights and shadows in the image. It also allowed me to ensure that the whites in the image were truly white and the black were truly black. I used the Lens Correction feature in darktable to accentuate the center of the image. I also retouched a few areas: an errant hair traversing one of the waves, a few bright pixels reflecting the LED in the middle of the pinch-off bulb, and a distracting shadow in the right upper corner that pulled the viewer's eye away from the jet. Finally, I enhanced the contrast and exposure using the RGB curve and Tone curve settings in darktable. You can find the Original and Edited image in the **Appendix** at the end of this document.

Conclusion

This image reveals a Worthington jet as it is about to pinch-off the upper portion. I appreciate that this image captures a brief moment in time. I also love that the surface of the water and the jet appear glassy, which heightens the drama illustrated within the image. To improve this image, I would experiment more with lighting. While I like the final result, I wish I had not had to retouch the top right corner of the image to get this result. I would also like to use a camera with a better lens that is also capable of faster shutter speeds. While my camera was sufficient, I think this image would have been easier to capture with a camera designed specifically for such high-speed imagery.

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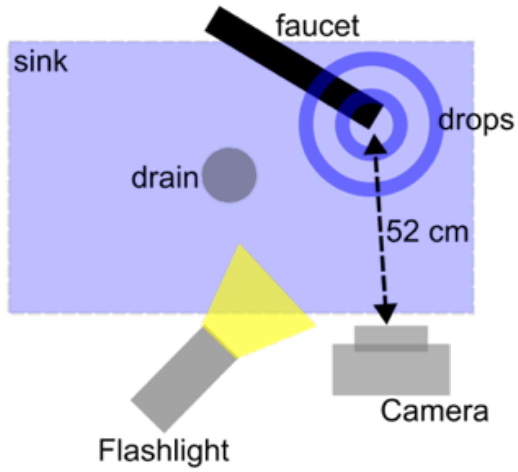
APPENDIX

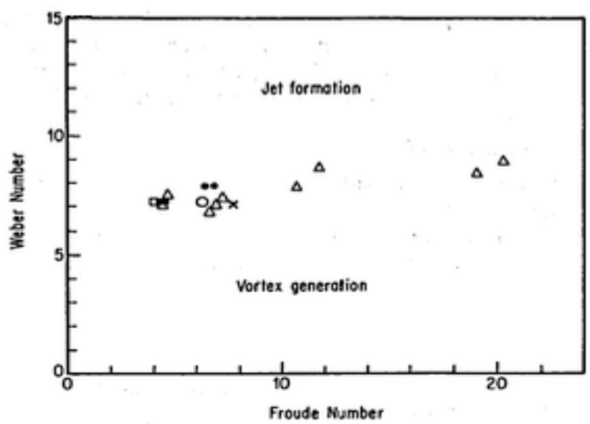
UNEDITED IMAGE

EDITED IMAGE

ANOTHER VIEW OF THE SETUP (OVERHEAD)



WEBER NUMBER VS. FROUDE NUMBER IN PREDICTING JET FORMATION



The relationship between the Weber number and the Froude number in predicting jet vs. vortex formation (Hsiao et al., 1988)