Radially Inward Fingering

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MCEN 4151: Flow Visualization, Get Wet

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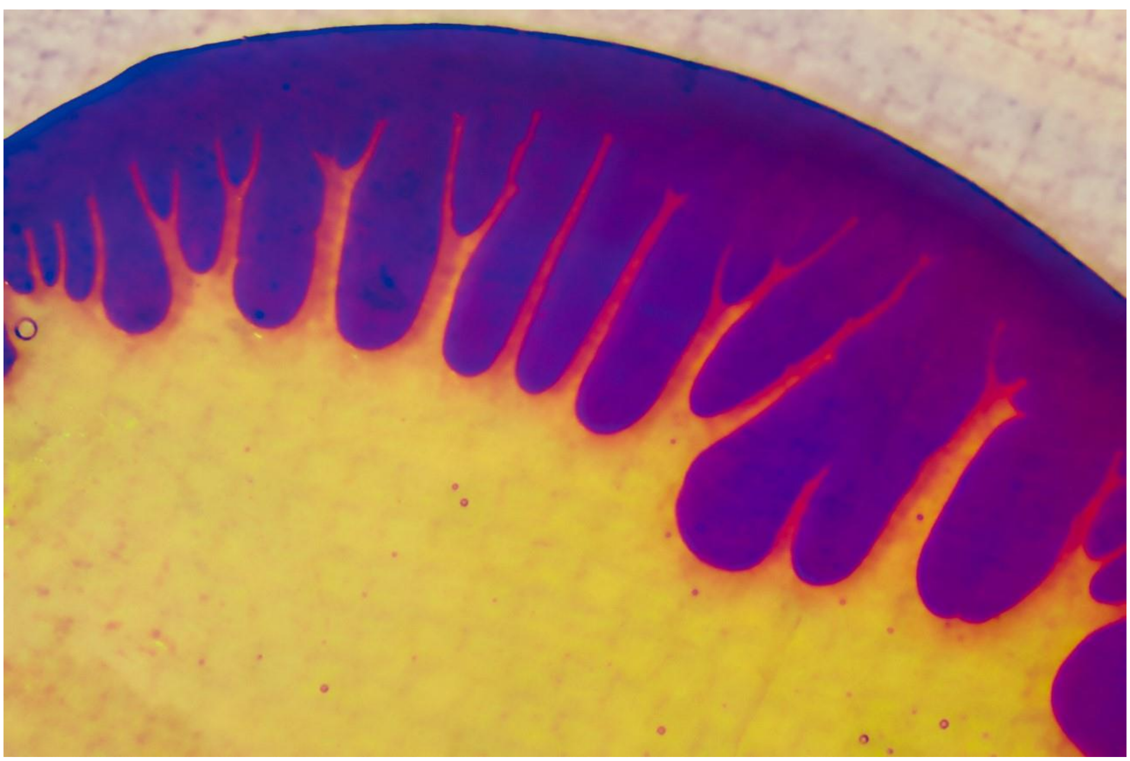

I. Introduction and Background

The intent of this image was to capture Saffman-Taylor instabilities, a fascinating flow phenomenon that occurs when a less viscous fluid flows through a fluid with higher viscosity. We achieved this effect through the use of a Hele-Shaw cell, made by cutting two square plates out of clear acrylic (B) and holding them together with clothespins. The more viscous fluid, in this case honey (C), was place between the two plates. A hole (F) was drilled through the top plate and a syringe (A) was used to inject the less viscous fluid. Several experiments were carried out in attempts to capture visually interesting Saffman-Taylor instabilities. This particular one was chosen for its uniqueness due to the direction of the flow. Generally, Hele-Shaw cells feature ST instabilities feature radially outward flows as the injection point is in the center of the plate. However, during this particular trial, the less viscous water was injected into a body of honey, where it produced typical ST instabilities until it exceeded the boundaries of the honey. Afterwards, the syringe was removed, creating a back pressure that pulled the water back into the honey. This created a series of Saffman-Taylor instabilities flowing inward through the honey. Special acknowledgements to Michael Becerra and Qisheng Lei for assistance in injecting fluids, cleaning the Hele-Shaw cell after each experiment trial, and taking many pictures.

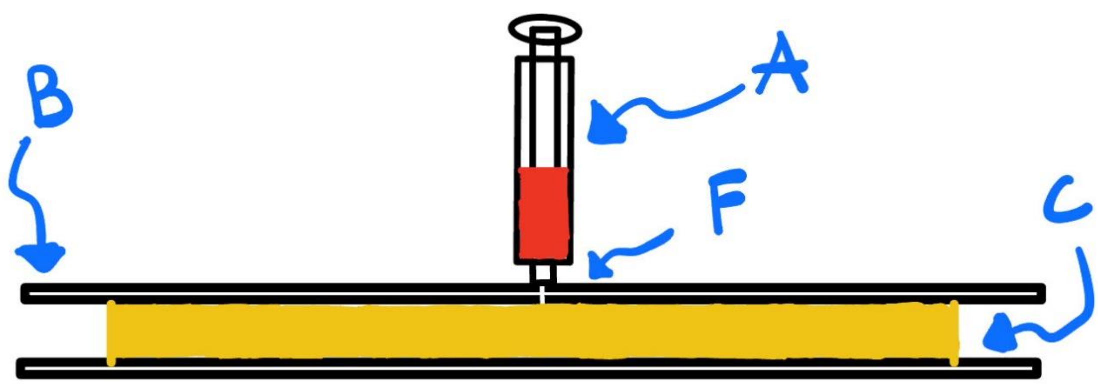

Fig 1: Experimental Setup (side view)

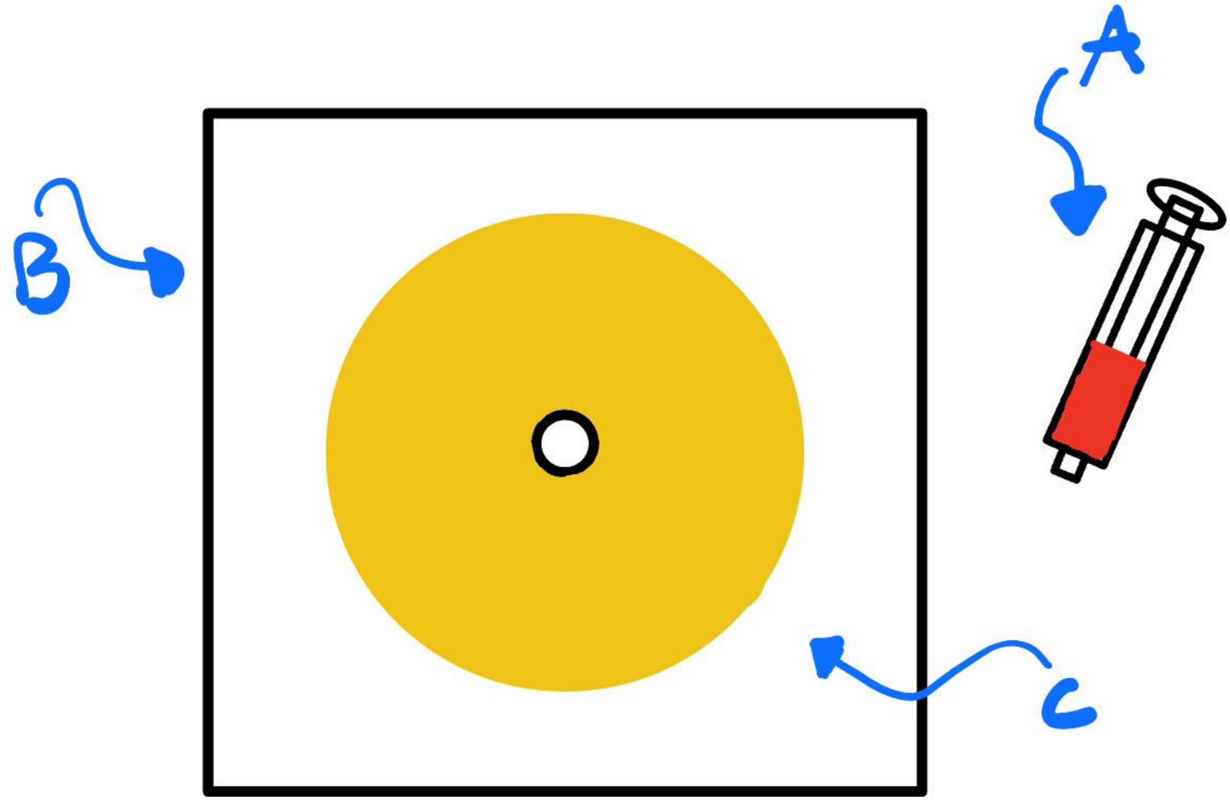

Fig 2: Experimental Setup (top view)

II. Experimental Setup and Theoretical Calculations

As mentioned and shown above, the experimental setup included a Hele-Shaw cell made out of two equally-sized square plates cut from acrylic. Clear acrylic was used in order to allow visualization of the flow and because it is widely available and cheap. These plates were roughly $1\ ft^2$. While the amount of honey applied was judged visually, the water was carefully measured to 10 mL for each trial. The plates were held together by clothespins after the honey was applied to the bottom plate, spreading the fluid out into a roughly circular shape centered around the insertion hole. This process left virtually no room between the plates, perhaps 1/64 in.

As the less viscous fluid expands, it is driven by a pressure from the syringe and resisted by the more viscous fluid's inertia and friction between both fluids and the plates. Naturally, the less viscous fluid will "seek" the path of least resistance and the fractal Saffman-Taylor instabilities are a reflection of this behavior. The fluid spreads in many different directions, favoring certain vectors that offer less resistance.

When quantifying Saffman-Taylor instabilities, a common technique is to calculate the growth rate, or the tendency of these instabilities to expand and split off into a multitude of directions, using Eqn 1.

$$\sigma = \frac{kV(\mu_2 - \mu_1) - \gamma H_f k^3}{\mu_1 + \mu_2} \quad \text{(Eqn 1)}$$

$Where\ \sigma = growth\ rate, k = wavenumber, V = velocity, \gamma = surface\ tension, H_f = mean\ curvature,$
$\mu_{1,2} = viscosities\ of\ fluids$

This equation features some high-level fluid dynamics and differential geometry concepts, but at first glance it can be seen that the growth rate is directly dependent on the difference in viscosities of the two fluids. Therefore, our initial experiments conducted using canola oil and water were fairly unsuccessful due to the relative similarity in viscosity. Henceforth, we switched to honey and water with much better results. The viscosities of these 3 fluids are shown in Table 1 below. As shown in the table, the difference in viscosity between water and honey is quite large, contributing towards a high growth rate resulting in the formation of a high quantity of "fingers."

Fluid	Dynamic Viscosity, μ $(mPa \cdot s)$
Canola Oil	46.2
Water	0.89 - 1
Honey	10000

Table 1: Dynamic Viscosity of Experimental Fluids

Another key quantity when studying Saffman-Taylor instabilities is the Reynolds Number, as laminar flow is essential in the formation of these fingers. Using Eqn 2, estimating the characteristic length as the distance between the plates at 1/64 in or 4.37 mm. The density of water is well known to be roughly 997 kg/m^3. While the flow speed was not explicitly measured, it is estimated as roughly 0.1 m/s^2 based on the size of the plates and how quickly the water spread outwards. With these values, we arrive at a Reynold number for water of roughly 435, well below the threshold to maintain laminar flow. With the incredibly small flow speed and distance between the plates, it is very easy to achieve laminar flow in a Hele-Shaw cell.

$$Re = \frac{\rho u L}{\mu} \quad \text{(Eqn 2)}$$

$Where\ \rho = density, u = flow\ speed, L = characteristic\ length,\ \mu = dynamic\ viscosity$

III. Visual Techniques and Camera Settings

The visualization techniques used in these experiments were quite simple. Honey naturally has a yellow hue to it so the water was simply dyed red to contrast the color of the honey. The cell was constructed of clear acrylic to allow easy insight into the flow behavior. The experiments were conducted in a well-lit area and surprisingly, glare on the plastic was never really an issue when taking photographs. No camera flash was used as the lighting in the CU Engineering Center's ITLL was plentiful, making it unnecessary. Additionally, camera flash would have likely caused significant glare on the top acrylic plate, obscuring the physics.

Table 2 below lists many of the camera's setting when taking the image. I intentionally used a large aperture to create a shallow depth-of-field and focus on the whole plate. When performing an experiment with large, flat plates, I wanted to create an image that did not distort this 2D plane. A high shutter speed was not incredibly important to this image since the flow was not particularly fast, so 1/100 was chosen to achieve a reasonable brightness and avoid blur from my hands shaking. The ISO was chosen automatically by the camera.

Camera	Canon EOS Rebel 1500D (Rebel T7)
Focal Length	55 mm
Aperture	f/5.6
Shutter Speed	1/100 sec.
ISO	ISO 2000
Dimensions	6020 x 4015 pixels

Table 2: Camera settings and image dimensions

The original image is shown below in Fig. 3. I really liked the fluid effects captured but I thought the color fell a bit flat and I wanted to frame the image to focus more on the Saffman-Taylor instabilities. When comparing Fig 3 to the final image in Fig 4, it can be seen that the image was cropped on a specific region of the phenomenon and that the color palette was altered through increasing the blue-yellow contrast. The saturation was also increased to make the image more vibrant.

Fig 3: Original (unedited) image

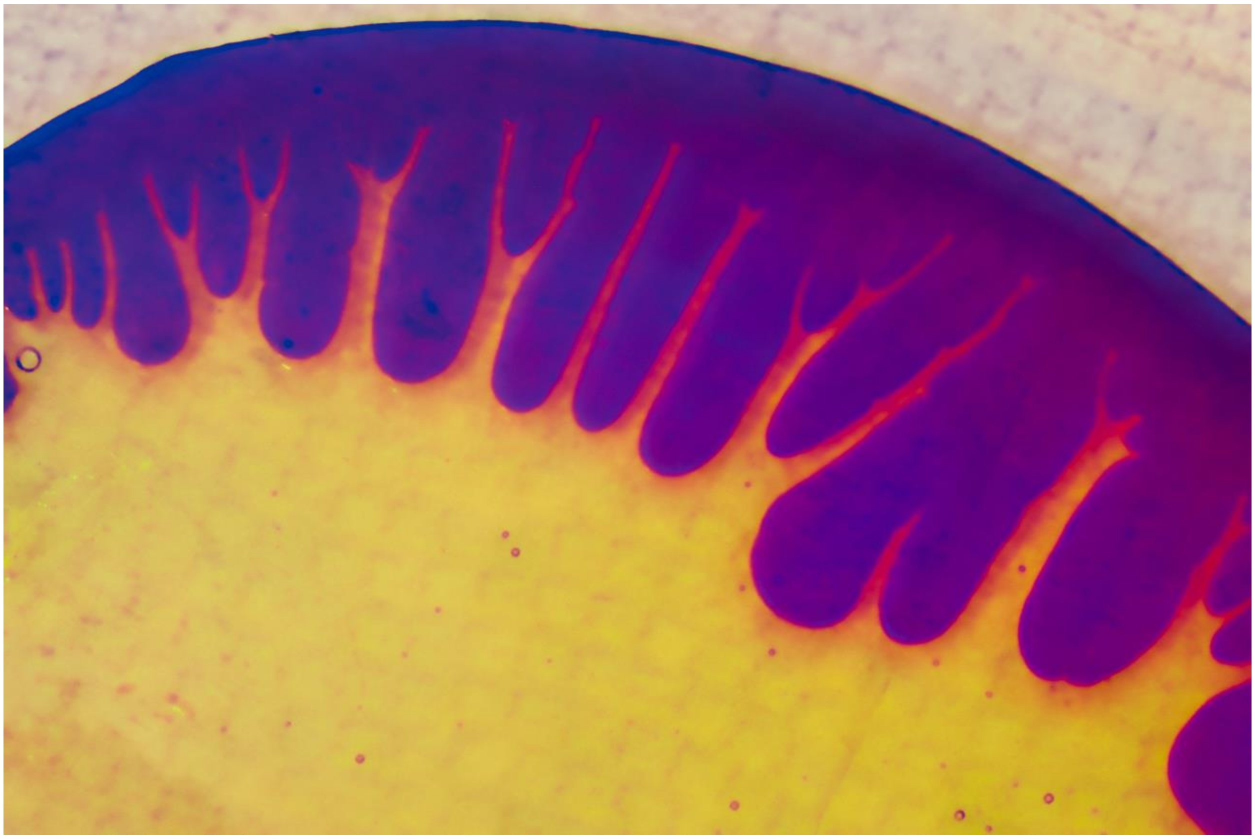

Fig 4: Final (edited) image

I really like this final image, I think it is vibrant and attention-grabbing with the color palette and high saturation. I like that at first glance, it appears that the honey is spreading outward in thin, vein-like tendrils, while in reality the water is spreading inward and these veins are regions between fingers of water. I was worried the physics was obscured by the tighter framing and the other editing choices, but I was told in my critique that the physics were still well highlighted. I am not sure what aspect of this particular image I would like to improve, but if I were to repeat this experiment, I think it'd be interesting to capture an image of this same effect but across the entire circumference of the honey region rather than just a small arc. While it was not initially my intent when planning this experiment to capture Saffman-Taylor instabilities propagating inward, once I saw them occur I knew I wanted to use an image of this effect. I think this image does a good job of displaying these physics in a unique, unorthodox context and is visually interesting without any context to viewers who know little about fluid mechanics.

References

Chart, Oil Viscosity. “Table of Viscosities.” *Viscosities of Common Fluids - Oil Viscosity Chart*, oilviscositychart.com/learn/viscosity-list.php. Accessed 6 Oct. 2023.

Numerical Simulation Based on the Volume-of-Fluid Approach ... - Iopscience, iopscience.iop.org/article/10.1088/1742-6596/1158/3/032036. Accessed 7 Oct. 2023.