Inverted Vortex Paradox

Team Second Report

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Abstract

This report details an experimental visualization of the inverted vortex paradox, also colloquially known as the tea leaf paradox, a fluid dynamic phenomenon where heavier particles or a denser fluid converge centrally in a rotating lighter fluid, defying expected centrifugal motion. The primary objective was to capture and analyze the inverted vortex that arises from secondary flow forces, and study vorticity—a ubiquitous yet complex feature in fluid systems. Conducted as part of the MCEN 5151 Flow Visualization course's Team Second assignment, the study aimed to elucidate the conditions fostering this enigmatic behavior in fluid dynamics. The study's findings contribute valuable insights into the fundamental principles of vorticity and secondary flow, with potential applications in atmospheric science, micro and nano particle aggregation, and numerous other fields as well.

Context and Purpose

Vorticity is a common and intriguing phenomenon in fluid dynamics, ubiquitous in nature and engineering yet complex in its manifestations. The primary objective of this photography assignment for the MCEN 5151 Flow Visualization course's Team Second assignment was to capture and analyze the phenomenon colloquially known as Einstein's tea leaf paradox. The purpose extended beyond mere observation to a deeper understanding of the conditions and variables that influence the formation of the counterintuitive inverted vortex arising due to secondary flow forces.

The captured image represents the observation of this phenomenon. This paradox occurs when denser particles in a fluid gather at the center of the vessel upon stirring, which is contrary to the expected centrifugal movement toward the vessel's edges. The purpose of capturing this phenomenon was to visualize the secondary flow effects in a two-fluid system, emphasizing the non-intuitive fluid dynamics at play.

Flow Apparatus

The experiment utilized a modified kitchen frother, typically operating at 3 volts, adjusted to approximately 1.1 volts to reduce its speed and suit the experimental requirements. This modified stirring mechanism created the precise conditions needed to induce the circular motion in the lighter liquid, giving rise to the inverted vortex for the denser liquid at their interface. The apparatus consisted of a 0.09 m diameter vessel, filled with a combination of blue-dyed distilled water and clear mineral spirit that created a distinct interface due to their immiscible nature and difference in density; mineral spirit being less dense, typically at approximately 790 kg/m^3. Only the upper lighter fluid was stirred using a modified kitchen frother that resulted in an inverted vortex, but stirring the lower denser liquid or both the liquids yields in normal vortices, which would hint at the fluid dynamics at play. The experiment was designed to be repeatable with consistent conditions with the same liquids being utilized for all the takes and the modified frother to stir the liquids too reach the same angular velocities. The flow observed showcases the secondary flow effect, where the boundary layer of the slower-moving lower liquid is pulled towards the center, thus creating an upward flow at the center instead of outward, resolving the counter-intuitive effect.

Visualization Technique

The visualization was achieved using blue dye in the distilled water and keeping the mineral spirit clear to aid in clear visualization of the denser water and the inverted vortex. A kitchen frother was modified by reducing the input voltage from the stated 3v to 1.1v in order to reduce the output rotational speed as well as aid in repeatability. A singular diffused light source of 10 cm diameter was utilized as a backlight, to provide direct and ample illumination to the fluid apparatus to counter the effects of using a very high shutter speed and small aperture. The surrounding environment was left dark to complement the direct lighting to generate dark, high contrasting borders on the liquid interface and bubbles. The light source was placed roughly 4 inches from the flow apparatus that was a glass jar. The camera was positioned approximately 2 inches from the jar and exactly in line to the light, optimizing the focal depth and contrast for the fluid phenomenon.

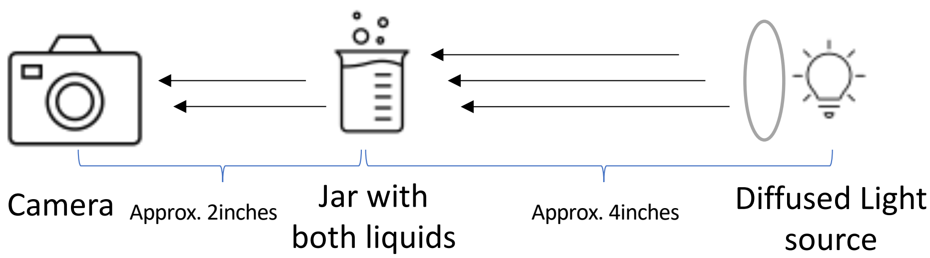

Figure 1 Photographic and Visualization Setup

Photographic Technique

The photographic equipment and settings were chosen to ensure the highest clarity and minimal motion blur while capturing the fast movement of the vortex. A Canon EOS 1500D camera equipped with an 18-55mm kit lens was utilized, set at a focal length of 55mm and the subject distance of around 4 inches to result in a 90mm wide frame of view. The camera settings were dialed to a high shutter speed of 1/2500sec to freeze the rapid motion, as well as high f-stops were employed to attain a broad depth of field, essential for capturing the entire depth of the fluid interface in sharp focus. Thus, an ISO of 6400 was needed to compensate for the reduced light due to the narrow aperture high shutter speed. Approximately 21 takes were necessary to adequately capture the image with the correct focus, lighting, and artistic intent, while still accurately defining the fluid phenomenon. Each attempt consisted of a continuous 10-shot burst over a 10-second timer, ensuring a range of images from which the optimal one was selected. All the images were captured in RAW format. For the final version, the original image resolution was 6000x4000, reduced to 5947x3958 post-editing on Darktable, which provided a platform for fine adjustments without altering the core visual data. Below is the final image and here is a link to a video of the flow: https://youtu.be/U7K0JGGR_Ok

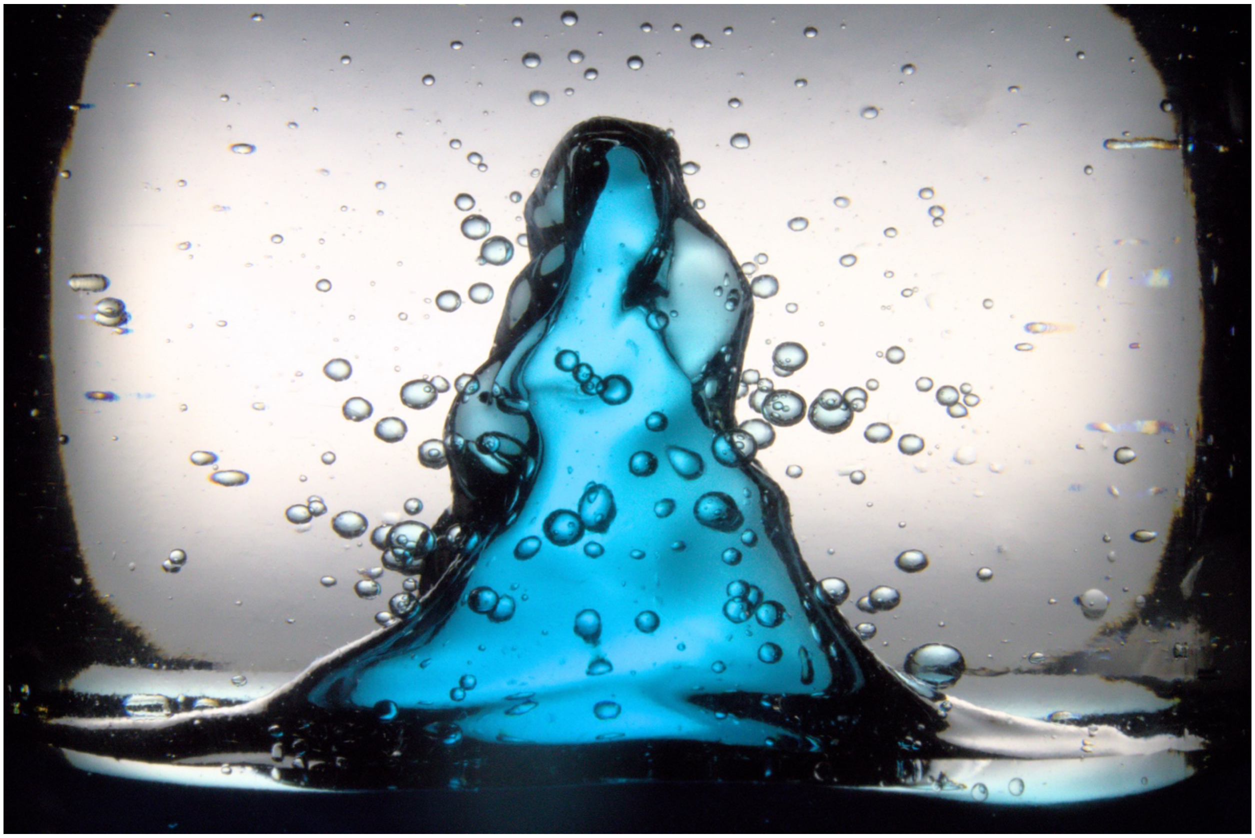

Figure 2 Final output image of the inverted vortex paradox

Fluid Dynamics

The inverted vortex paradox is a captivating example of secondary flow in action[1], where the central convergence of denser particles in a rotating fluid is driven by underlying vorticity and pressure gradients[2]. The forthcoming analysis and calculations indicate the flow in the clear mineral spirits to be turbulent which indicates chaotic changes in the pressure and flow velocities, the interaction of which at the fluid interface gives rise to the counterintuitive behavior of the inverted vortex paradox. Noting that the inverted vortex only arises when the upper lighter liquid is stirred, an easy-to-understand explanation of this would be that as the upper liquid is stirred, it introduces inertial forces in the liquid and pushes it outwards towards the wall and creates a pressure gradient with the least pressure in the center and highest pressure near the wall of the vessel, which normally results in the liquid to creep up the container's walls and creates an ordinary vortex at the air and upper liquid interface, with a dip in the center. However, for two liquids, the low pressure in the center and high pressure at the outer edge in the upper liquid and at the interface results in the bottom heavier liquid to rise up in the center and create an inverted vortex. This experiment aimed to visualize these secondary flows, which are often overshadowed by the primary flow but are integral to the transport and mixing of fluids. The precise control over the stirring mechanism's speed was essential to replicate the delicate balance of forces necessary for the paradox's emergence.

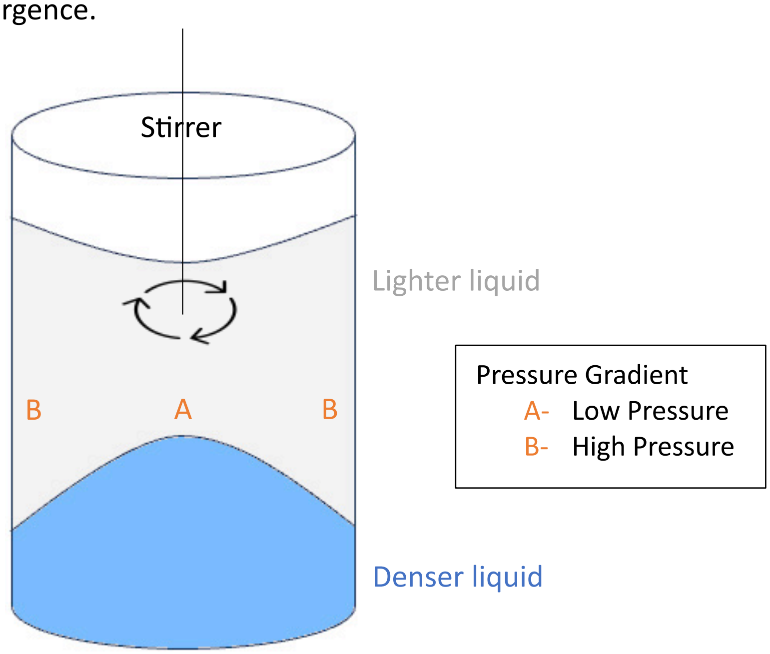

Figure 3 Illustration of Pressure Gradient in Apparatus

Scientific Analysis and Calculations

The tangential velocity at the vessel's wall, essential for vortex formation, was determined by first calculating the angular velocity. The bubbles completed one revolution, 2π radians, in approximately 0.1s. So the angular velocity can be calculated as:

$$\omega = 2\pi/t = 2\pi/0.1 \approx 63\text{rad/s}$$

The tangential velocity, v_t at the walls of the 0.09m diameter cylindrical container can thus be calculated as:

$$v_t = \omega \times r$$
$$v_t = 63 \text{ rad/s} \times 0.045 \text{ m} \approx 2.835 \text{ m/s}$$

Bernoulli's principle was applied to estimate the pressure difference driving the secondary flow and resulting vortex of approx. height 3cm:

$$\Delta P = \rho \times g \times h - \tfrac{1}{2} \rho \times v_t^2$$
$$\Delta P = 1000 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \times 0.03 \text{ m} - \tfrac{1}{2} \times 1000 \text{ kg/m}^3 \times (2.835 \text{ m/s})^2$$

$$\Delta P \approx -3724.31 \text{ Pa}$$

This negative pressure difference, integral to the inward movement of the denser fluid, aligns with the secondary flow phenomena described in "Mysteries of Engineering Fluid Mechanics" by Gordon D. Stubley[3].
Additionally, the Reynolds number, a dimensionless quantity used to predict flow patterns, was calculated as follows:

$$Re = (\rho \times v \times L) / \mu$$
$$Re = (1000 \text{ kg/m}^3 \times 2.835 \text{ m/s} \times 0.09 \text{ m}) / 0.001 \text{ Pa·s} \approx 127320$$

This high Reynolds number indicates a turbulent flow regime, suggesting that the fluid motion is characterized by chaotic changes in pressure and flow velocity. Turbulence significantly affects the mixing and transport properties of the fluid, enhancing the interaction between different fluid layers and thereby influencing the formation and stability of the inverted vortex. The turbulent nature of the flow contributes to the unpredictable and counterintuitive behavior observed in the tea leaf paradox.

Conclusions and further directions:

The experiment successfully visualized the tea leaf paradox, highlighting the secondary flow's role in fluid dynamics. The study exemplifies the importance of vorticity and pressure gradients in influencing fluid motion and encourages further exploration into the impact of fluid properties and rotational forces on vortex behavior. This research not only serves as an educational tool in flow visualization but also as a stepping stone for future fluid dynamics investigations. The exploration of the inverted vortex paradox within this experiment has provided insights into the intricate nature of secondary flows. Future investigations could consider the influence of vessel shape and fluid properties on vortex formation, informed by studies on secondary flow mechanisms within various engineering systems. These secondary flows and particularly the inverted vortex phenomenon can be applied in atmospheric science, micro and nano particle aggregation, and numerous other fields as well.

References

[1] P. Bradshaw, "Turbulent Secondary Flows," *Annu. Rev. Fluid Mech.*, vol. 19, no. 1, pp. 53–74, 1987, doi: 10.1146/annurev.fl.19.010187.000413.

[2] D. F. Long, S. V. Perivilli, and J. W. Mauger, "Einstein's tea leaf paradox and its relevance to dissolution testing," *Dissolution Technol.*, vol. 21, no. 3, pp. 17–19, Aug. 2014.

[3] G. Stubley, "Mysteries of Engineering Fluid Mechanics," 2001. Accessed: Nov. 23, 2023. [Online]. Available: https://www.semanticscholar.org/paper/Mysteries-of-Engineering-Fluid-Mechanics-Stubley/92699ff8edb6c840b92fb9f8169a5a7bdff1f068