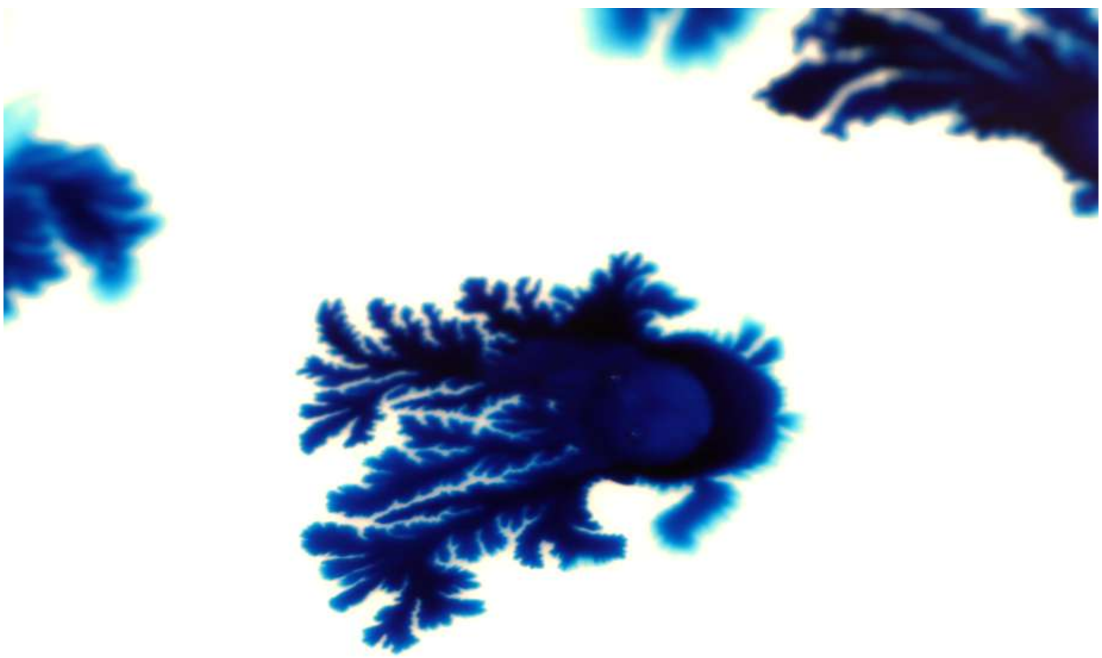
Saffman-Taylor Instabilities



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Get Wet Assignment Report

MCEN 5151: Flow Visualization

September 24th, 2025

Introduction

The Viscous Fingering image was created as the first assignment in a flow visualization class. The goal of the image was to capture a pattern formation that occurs when a low viscosity fluid moves through a higher viscosity fluid. The differing viscosities produce a phenomenon known as Viscous Fingering, or the Saffman-Taylor instability, and produce intricate branching structures that make fluid physics easy to observe. I performed the setup and execution of this project alone. The intent of the image was to capture the Saffman-Taylor instabilities and fingering to observe how viscosity, surface tension, and flow instabilities affect the flow. The artistic intent of the image was to show intricate patterns that appear like art. The final image brings out fascinating instabilities that can be amplified with the use of blue food dye.

Setup

The apparatus used to capture the flow was a stationary paper plate sitting horizontally on a table. The base fluid was a thin layer of white Elmer's glue approximately one fifth of an inch deep. The glue covered the entire bottom surface of the paper plate.

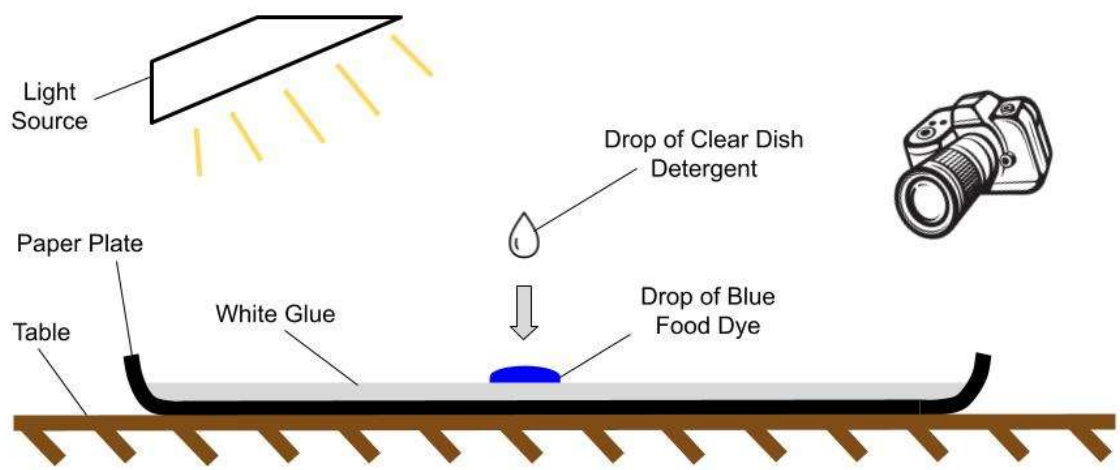

Figure 1. Flow apparatus setup diagram

Several drops of blue food dye were then dropped on the plate. At this time the dye stays in its drop formation. For a sense of scale, the drop of food dye is about 1.2 centimeters in diameter before the detergent is added. Finally, a small drop of clear liquid dish detergent was added on top of each drop of food dye. For this experiment, Dawn odorless dish detergent was used. A small droplet (less than half a centimeter in diameter) of detergent is all that is needed. The food dye will stay in the droplet shape for several minutes, however, adding the detergent droplet quickly after was observed to produce better fingering. The addition of dish detergent commences the formation of the fingers. Instabilities start to form as the dye and detergent mixture moves through the white glue.

Flow Physics

The flow observed in this image is an example of displacement flow. In this case, a less viscous fluid flows through a more viscous medium. At first, the drop of blue dye alone is held together by surface tension, and can only diffuse through the glue at a very slow rate. When the very low viscosity dish detergent is added to the dye droplet, the viscosity in the mixture becomes significantly less than this viscosity of the glue. This causes the flow to evolve and spread outward driven by both contrasts in viscosity and gradients in surface tension. We will calculate several dimensionless parameters to check if the flow is driven by both viscosity and surface tension.

The first dimensionless number we will investigate is the Reynold's Number. We will estimate the Reynold's number to approximate the ratio of inertial forces to viscous forces in the flow. A low Reynold's number indicates that viscous forces dominate the flow. Before computing this parameter, we need a sense of scale of the experiment:

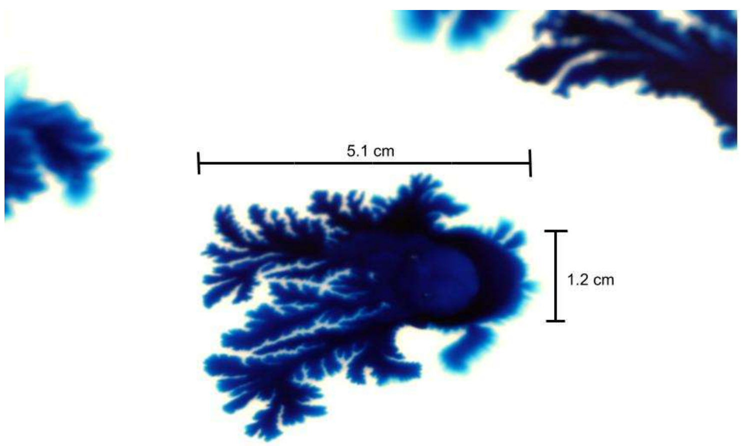

Figure 2. Reference dimensions for estimating flow physics parameters.

As seen in figure one, the droplet we will be referencing has an initial diameter of 1.2 centimeters, then flows to a complete distance of 5.1 centimeters.

The Reynolds number is defined as:

$$Re = \frac{\rho U L}{\mu}$$

Where ρ is the density of the flowing fluid, U is the flow velocity, L is the characteristic length of the flow and μ is the dynamic viscosity of detergent. We will approximate the density of the detergent similar to a typical aqueous solution, $1000\ \frac{kg}{m^3}$. We can approximate the flow velocity as $2 \cdot 10^{-3}\ \frac{m}{s}$ since we know it took approximately 30 seconds for the detergent to flow 5.1 centimeters. The characteristic length of the flow is $0.051\ m$, and the viscosity of a typical dish detergent is $0.4\ \frac{Pa}{s}$.[1] Plugging in our values to estimate the Reynolds number in our flow:

$$Re \approx \frac{(1000\ \frac{kg}{m^3})(2 \cdot 10^{-3}\ \frac{m}{s})(0.051\ m)}{0.4\ \frac{Pa}{s}} = 0.255$$

The resulting Reynolds number, $Re = 0.255$, is extremely low, which confirms that inertial forces are negligible compared to the viscous forces. The low Reynold's number show that viscous forces dominate the flow and we can neglect inertial forces.

The second dimensionless number we will investigate is the viscosity ratio. This parameter will tell us if the difference in fluid viscosities will be significant. The viscosity ratio is defined as:

$$M = \frac{\mu_{displaced}}{\mu_{displacing}}$$

In our case we will use the above viscosity value for the detergent viscosity, $0.4\ \frac{Pa}{s}$. For the viscosity of glue, we will reference a typical glue viscosity of $100\ \frac{Pa}{s}$.[2] Plugging in our experiment values in the viscosity ratio equation:

$$M \approx \frac{100\ \frac{Pa}{s}}{0.4\ \frac{Pa}{s}} = 250$$

A large viscosity ratio where $M \gg 1$ is the canonical condition for strong Saffman-Taylor fingering, which indicates that a low viscosity fluid penetrating a high viscosity fluid promotes interfacial instability and slender fingers.[3]

The third nondimensional number we can reference to confirm our flow physics is the capillary number, which relates the viscous and capillary forces in the flow. The capillary number is given by:

$$Ca = \frac{\mu U}{\gamma}$$

For our estimations, we can use the above viscosity of dish detergent and the estimated flow velocity. The surface tension, γ, of pure water (approximately the same as food dye) is $0.072\ \frac{N}{m}$, but surfactant solutions such as dish soap will typically reduce this surface tension to approximately $0.03\ \frac{N}{m}$.[4] Plugging in our estimated values we get:

$$Ca \approx \frac{(0.4\ \frac{Pa}{s})(2 \cdot 10^{-3}\ \frac{m}{s})}{0.03\ \frac{N}{m}} = 0.0267$$

This relatively small capillary number indicates that viscous stresses are significant, but capillary forces are important on smaller scales. The tip-splitting and secondary branching observed in the flow indicate that the viscous forces are driving fingers outwards, while surface tension (capillary forces) controls the finger width and tip stability.

The analysis of three dimensionless parameters in the flow shows us three important indicators in the flow experiment. The low Reynold's number indicates that the flow is driven by viscous forces and not inertial forces. The large viscosity ratio indicates there are strong Saffman-Taylor instabilities occurring, which occurs when a less viscous fluid displaces a more viscous fluid. Finally, the capillary number indicates that both viscous and capillary forces are present, but surface tension affects the flow more at smaller scales.

Visualization

In order to visualize the fluid physics and instabilities described above, several different visualization techniques were used. The blue food dye acts as a tracer for the fluid, and merely mixes with the detergent to show flow patterns. Any store bought liquid food coloring works for this visualization, but certain colors may have different instability effects. It helps if the food dye comes in dropper bottles for easy placement of singular drops. Both the food dye and detergent were not diluted in any way. Lighting was also an important aspect of visualization in this experiment. Four LED kitchen spotlights were pointed directly at the paper plate approximately one and a half meters above the table, along with one desk lamp illuminating the experiment from 40 centimeters away. The ample lighting in the setup allowed for great contrast between the white glue and blue food dye, which properly showed the intricate branching structures.

Photography and Post-Processing

The image was captured using an EOS R100 digital camera. The field of view was approximately half the size of the paper plate, or 20 by 15 centimeters. The camera was positioned about 25 centimeters from the paper plate, at an angle of about 40 degrees from the horizontal table. A focal length of 22 mm was used. The exposure setting used were an aperture of F7.1, a shutter speed of 1/8, and an ISO of 100. With the shutter speed of 1/8, and the fluid velocity of $2 \cdot 10^{-3}\ \frac{m}{s}$, the flow moves approximately 0.25 millimeters during the exposure, which is an insignificant distance when considering blur and the scale of the image. The original image was 6000 by 4000 pixels in size. No flash was used, as the lighting setup described above was providing sufficient lighting.

Post-processing and editing of the image were aimed at emphasizing the fluid physics observed, and allowing the viewer to focus on the intricate fingering patterns. All image editing was done in Dark Table.

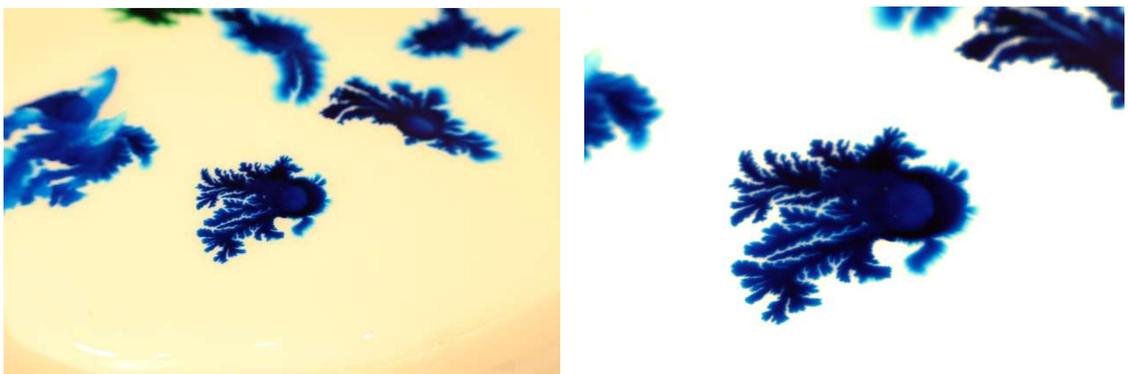

Figure 3. Side by side images of unedited image (left), and final image (right)

A crop was performed to focus on one droplet instability pattern, which two more patterns in the background to give the image a sense of depth. The image size after cropping was 1300 by 900 pixels. Next, the yellow tint of the glue was removed by reducing the brightness of yellow in the color zones tool. Also using the color zones tool, the blues were enhanced by increasing the blue lightness. These coloring edits increase the contrast between the glue and blue fingering patterns which better emphasizes the flow phenomena.

Conclusion

The final image reveals the intricacies and fractal-like growth of the Saffman-Taylor instabilities. The image clearly shows instabilities as the flow expands radially due to differences in viscosities in the two fluids. One thing I really enjoy about this image is the contrast of the deep blue colors with the white glue, which allows the viewer to see every detail of the fingers. One aspect about the image that I would like to improve is to have the entire droplet and instability in focus. While one section of the droplet is in focus, it seems that others are out of focus. The image fully satisfied my intent, as it properly shows the fluid physics of the Saffman-Taylor instability and is visually appealing. In future experiments, it would be interesting to see how fingers from two separate dye droplets react when they come in contact with each other.

References

[1] Materials Education (n.d.). *Viscosity of household fluids.* Materials Education. https://materialseducation.org/educators/matedu-modules/docs/viscosity_of_household_fluids.pdf

[2] Powerblanket. (2023, March 28). *Sticking to the right consistency: How temperature influences industrial glue viscosity and storage.* Powerblanket. https://www.powerblanket.com/blog/sticking-to-the-right-consistency-how-temperature-influences-industrial-glue-viscosity-and-storage/

[3] Saffman, P. G., & Taylor, G. I. (1958). *The penetration of a fluid into a porous medium or Hele-Shaw cell containing a more viscous liquid. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences.*https://royalsocietypublishing.org/doi/10.1098/rspa.1958.0085

[4] Shen, A. Q., Gleason, B., McKinley, G. H., & Stone, H. A. (2001). *Fiber coating with surfactant solutions. Physics of Fluids, 13*(1), 20–27. https://www.princeton.edu/~stonelab/Publications/pdfs/From%20Howard/PhysicsFluids/ShenGleasonMcKinleyStonePhysFluids.pdf