

UNIVERSITY OF COLORADO - BOULDER

MCEN 5151
FLOW VISUALIZATION

Get Wet: How to Reflect a Cannon Ball

Authors:
Bryce Pfuetze

18 September 2026



College of Engineering & Applied Science
UNIVERSITY OF COLORADO **BOULDER**

I Introduction

The first assignment for the Flow Visualization course is called "Get Wet", as a way to get your feet wet with photography and fluids flows. I took this title literally as I made my friend Ben Gebhart jump into a pool a dozen times on a cloudy day.

In my first attempts, I simply took photos from the side of the pool, capturing the splash that resulted when he jumped in. This assignment had the added flexibility to use any camera, I wanted to take advantage of this by using a waterproof GoPro. My hope was to place the camera at the surface of the water so half the frame could see under water and the other half above. Seeing the same splash through two different mediums seemed like an interesting way to illustrate differences in refractive indices. The waves in the pool were large when compared the the camera, and it was almost impossible to get a photo split in the middle like this. The lens would either be entirely submerged or completely out of the water. When I began reviewing these photos I noticed the reflection effect at the boundary between air and water. I began to pursue a photo that instead showed a jump into a pool with its reflection in the other half of the frame.

II Test Setup

The subject is approximately 2 meters away, 1 meter below the surface of the water, and the camera is about 5 centimeters (0.05m) below the surface. A small salt water pool was used with a depth of 6 feet where the photo was taken.

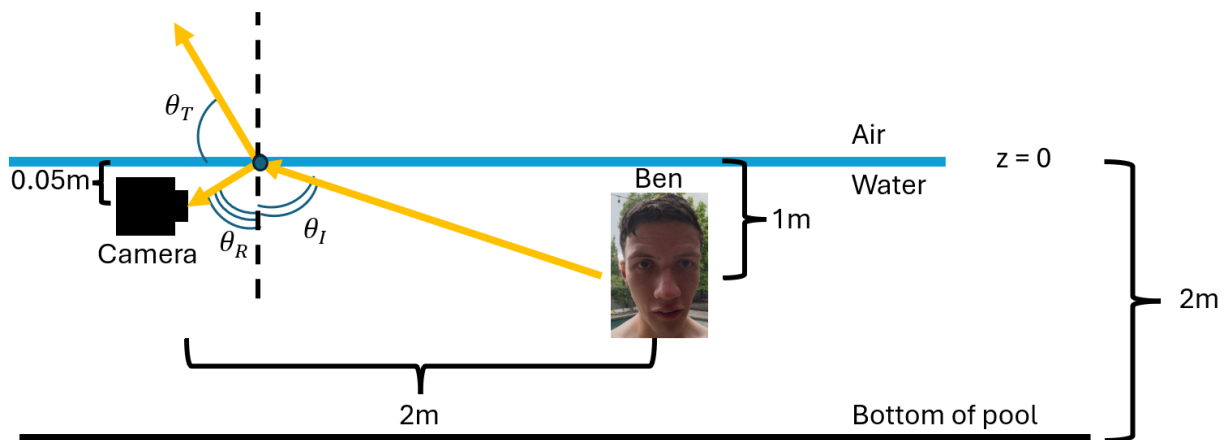


Figure 1: Experiment Setup

III Flow Phenomena

In class we discussed light refraction as it passes from one medium to another, like from air to water. This shot takes advantage of the other half of Snell's Law: reflection.

To understand Snell's law we will follow the derivation in Haberman [1], which begins with the three dimensional wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (1)$$

With solutions taking the form of

$$A = e^{i(\vec{k} \cdot \vec{x} - \omega t)} \quad (2)$$

where k is the wave vector in 3 components. The incoming wave is called the Incident Wave and is denoted with the subscript I . A component of this wave is reflected at the interface, this is the Reflected Wave R . Finally, we assume that a component is refracted in the second medium at $z < 0$ as the Transmitted Wave T . The wave frequency is known to be $\omega^2 = c^2 k^2$ for all three dimensional waves.

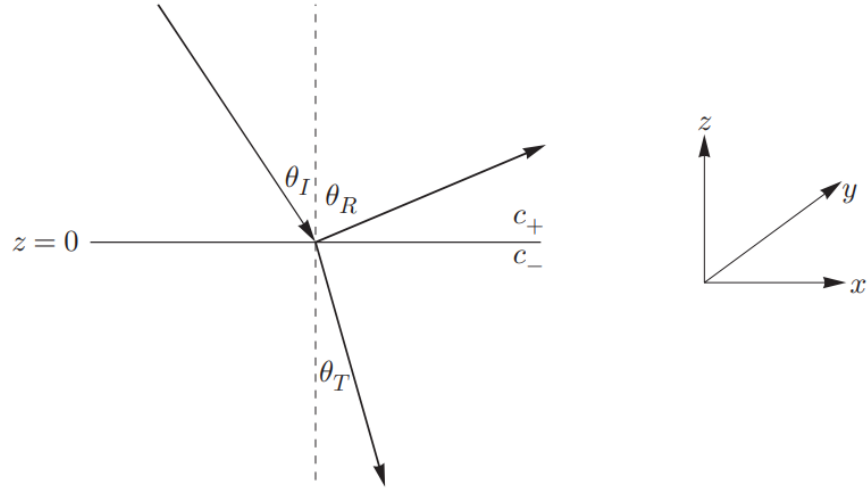


Figure 2: Reflected and refracted (transmitted) waves from Haberman [1]

One of the boundary conditions that must be enforced is continuity at the interface between the two mediums. This requires that the frequency for the incident, reflected, and transmitted waves all be equal:

$$\omega_+(k_I) = \omega_+(k_R) = \omega_-(k_T) \quad (3)$$

Note that two of these waves are in the same medium, meaning they have the same speed c . Given the definition for ω we know that k for both the Incident and Reflected waves must also be the same. This requires that, with respect to the interface normal vector, the angle of reflection be equal to the angle of incidence. The incident and transmitted waves can then be related by:

$$k_I \sin(\theta_I) = k_T \sin(\theta_T) \quad (4)$$

This finally leads us to Snell's law, which is as follows:

$$\frac{\sin \theta_T}{\sin \theta_I} = \frac{k_I}{k_T} = \frac{c_-}{c_+} \quad (5)$$

We can now estimate the path for light leaving our subject and entering the camera. The approximate values shown in Section II will be used. What angle would light need to leave the subject at to reach the camera sensor? What angle does the light get refracted at? Using some trigonometry the angle of incidence is found to be $\theta_I = 62.30^\circ$, which is also θ_R . Now lets solve for θ_T :

$$\begin{aligned} \frac{\sin \theta_T}{\sin \theta_I} &= \frac{k_I}{k_T} \\ \theta_I = 62.30^\circ, k_{water} &= 1.34 \text{ and } k_{air} = 1.0003 \\ \frac{\sin \theta_T}{\sin 62.03^\circ} &= \frac{1.34}{1.0003} \\ \sin \theta_T &= 1.19 \end{aligned}$$

This is not possible, no angle will allow sin to be greater than 1. What is going on here? Light exhibits some very interesting behaviors when the two mediums are air and water, specifically when viewed from the water side. The first, which cannot be seen here but is quite interesting, is Snell's window. This happens when the observer is well below the surface and looks up. There will be a circle of light the observer can see with darkness surrounding it. The observer has a cone of approximately 90° visible to them because light outside of this cone is refracted away. This can lead to some very fun effects with a subject in the middle of this circle of light. The other, which we have encountered here, is Total Internal Reflection.

Total Internal Reflection requires light to travel from a material of higher refractive index to one of a lower refractive index, like glass to air or water to air. This can also be described as a more dense to less dense material or one with slower speed of light to faster.

We have shown a case that requires, for sufficiently large θ_I , that $\sin(\theta_T)$ be greater than 1. This is not possible, requiring instead that all of the incident wave be reflected. We can calculate this critical angle for air and water:

$$\begin{aligned} \frac{\sin \theta_T}{\sin \theta_I} &= \frac{k_I}{k_T} \\ \sin(\theta_T) = 1, \theta_I = \theta_{critical}, k_{water} &= 1.34, \text{ and } k_{air} = 1.0003 \\ \frac{1}{\sin(\theta_{critical})} &= \frac{1.34}{1.0003} \\ \theta_{critical} &= 48.29^\circ \end{aligned}$$

This means light traveling from the subject at an angle of greater than 48.29° from the vector normal to the water surface will get reflected rather than having some component refracted into the air. So this means the incident rays from the subject we looked at before would get reflected entirely back into the water.

Total Internal Reflection has many research applications. For example, Prieve and Frej [2] showed how it could be used as a non-intrusive and instantaneous way to measure separation

distance. They were able to use this to measure changes as small as 1 nanometer. Another application is in Biology, where it can be used at the solid and liquid boundaries of cells, as shown by Axelrod, Burghardt, and Thompson [3]. It has been used to measure adsorption (the adhesion of molecules to a liquid or solid surface, cool!) and observe cellular contact regions.

Finally, we should estimate the Reynolds number for this experiment. The fluid in this case is at rest, but the body is moving relative to it, much like a wing cutting through air. We estimate that Ben's center of mass starts at 1 meter above the pool at rest. It is accelerated at 9.8m/s^2 until it hits the pool surface. We take the characteristic length to be Ben's height, 2 meters, which is also the depth of the pool:

$$\begin{aligned}
 U^2 &= v_i^2 + 2a\Delta x \\
 U &= \sqrt{0 + 2(9.8)(1)} \approx 4.43\text{m/s} \\
 Re &= \frac{UD}{\nu} \\
 Re &= \frac{4.43\text{m/s} * 2\text{m}}{1.004 * 10^{-6}\text{m}^2/\text{s}} \approx 8820000
 \end{aligned}$$

This is well above the turbulent threshold for an external flow, which we can clearly see in the bubbles being carried downwards with Ben.

IV Visualization Technique

The total reflection effect is reduced by light sources from outside the water shining through from above. This could be improved if shot when it is dark out with only under water light sources. At the time of the photo there was significant cloud cover, which at least diffused the sunlight. The pool used was salt water, leading to a very slightly higher refractive index compared to $k = 1.333$ for pure water. No tracers were used but plenty of bubbles were introduced when the subject jumped in.

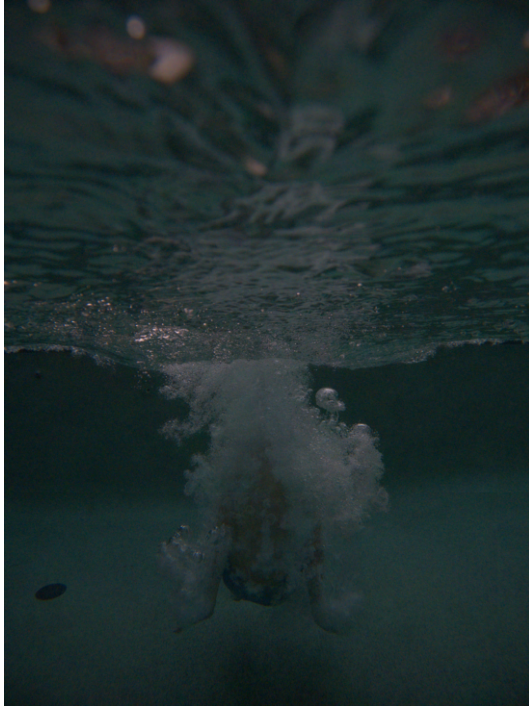
V Photographic Technique

Many of the settings in this shot were automatically chosen by the camera. I set it to capture a burst of 10 photos over 1 second after the shutter button was pressed. I used the waterproof housing and was, surprisingly, able to set it to capture in RAW + JPG. Some of the photos in a burst would come out over or under exposed, it is interesting how significantly it was changing the settings in between shots in the burst.

The final photo was edited in DarkTable. The heal tool was used to remove various drains along the pool. The biggest parts I changed, and those I was most unfamiliar with, were white balance and color calibration. The original image was much too green, but I was able to turn it more towards white and blue in the water.

Camera	GoPro Hero 10	FOV	121° (vertical)
Type	Digital Action	Distance to Object	≈ 2 meters
Original Image	4176x5568 pixels	Aperture	f/2.5
Final Image	4176x5568 pixels	Shutter Speed	1/964 second
Lens	Fixed + Waterproof shell	ISO	437

Table 1: Photo Specifications



(a) Original RAW Image



(b) Edited Image

Figure 3

VI Conclusion

The image reveals some more static fluid properties with a dynamic subject. Wind created a choppy surface to the water, I would have liked to capture something smoother before the ripples from the dive radiated outwards so that the surface acted more like a mirror. When originally taking photos above the water I was wishing for more sunlight, but after going this direction I think I should have waited until dark with some underwater light sources. Overall I am happy with the photo I ended up with for this first assignment, but have ideas I would like to try another time.

References

- [1] Richard Haberman. *Applied Partial Differential Equations: With Fourier Series and Boundary Value Problems*. 5th. Boston: Pearson, 2013. ISBN: 978-0321797056.
- [2] Dennis C. Prieve and Nasser A. Frej. “Total internal reflection microscopy: a quantitative tool for the measurement of colloidal forces.” In: *Langmuir* 6.2 (Feb. 1990), pp. 396–403. ISSN: 0743-7463. DOI: [10.1021/la00092a019](https://doi.org/10.1021/la00092a019). eprint: <https://pubs.acs.org/langd5/article-pdf/6/2/396/2527814/la00092a019.pdf>. URL: <https://doi.org/10.1021/la00092a019>.
- [3] Daniel Axelrod, Thomas P. Burghardt, and Nancy L. Thompson. “Total Internal Reflection Fluorescence.” In: *Annual Review of Biophysics and Bioengineering* 13.1 (1984), pp. 247–268. DOI: [10.1146/annurev.bb.13.060184.001335](https://doi.org/10.1146/annurev.bb.13.060184.001335). URL: <https://doi.org/10.1146/annurev.bb.13.060184.001335>.