

Chladni Patterns: Sand on a Vibrating Plate

Team First — Flow Visualization

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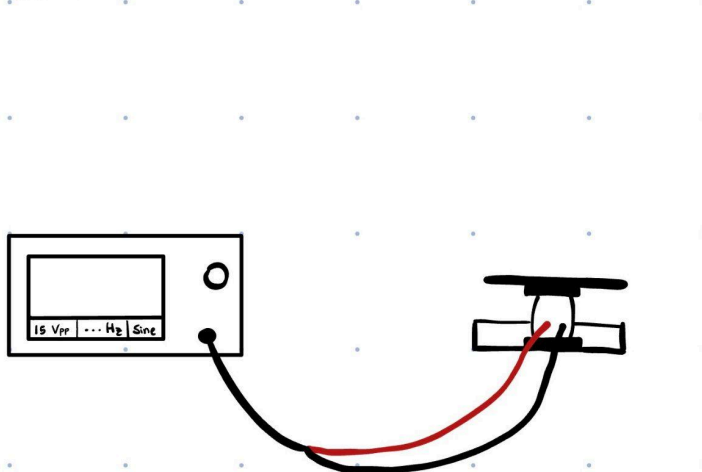
I.

For our first team assignment, our group recorded a video of sand on a vibrating plate. Our goal was to visualize how changing the vibration frequency creates different patterns in the sand. By testing several frequencies, we wanted to capture both the movement of the sand and the shapes that formed. The vibrations move the sand away from parts of the plate that shake more, causing it to collect along lines where there is very little movement. The resulting shapes are called Chladni patterns.

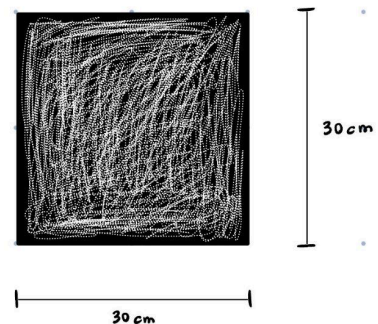
II.

The experimental setup is shown in the figure below. It consists of a plate with sand spread across its surface, with an oscilloscope connected to measure and display the electrical driving signal. The plate's dimensions are 30×30 cm. We used a sinusoidal signal with a peak-to-peak driving voltage of 15 V and tested frequencies of 209, 350, 505, 825, 1111, 1443, and 1774 Hz. As we changed the frequency, the sand moved across the plate and formed different patterns, which we recorded on video.

SIDE VIEW



TOP VIEW



The sand pattern is a picture of the plate's vibration mode, so the physics starts from the plate's equation of motion. Unlike a drumhead, which obeys the 2-D wave equation because tension restores it, a metal plate is restored by bending stiffness and obeys the Kirchhoff-Love equation,

$$D\nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} = 0, \quad D = \frac{Eh^3}{12(1-\nu^2)}$$

(Gander and Kwok 2012; Tuan et al. 2015b), where w is the transverse displacement, E is Young's modulus, ν is Poisson's ratio, ρ is the density, h is the thickness and D is the flexural rigidity. For steady driving at frequency f the plate settles into a standing wave,

$$w(x, y, t) = W(x, y) \cos \omega t, \quad \omega = 2\pi f$$

which reduces the problem to a spatial eigenvalue problem for the shape W ,

$$\nabla^4 W = k^4 W, \quad k^4 = \frac{\rho h \omega^2}{D}$$

Separation of variables gives a Fourier series of standing waves, which for a simply supported square plate of side L is

$$w(x, y, t) = \sum \sum a_{mn} \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{L} \cos \omega_{mn} t$$

And where the solved natural frequencies are

$$\omega_{mn} = \left(\frac{\pi}{L}\right)^2 \sqrt{\frac{D}{\rho h}} (m^2 + n^2)$$

In which each mode is excited in proportion to its shape value at the driver and amplified near its resonance, so a mode with a node at the center bolt cannot be excited at all. The plate thickness and material were not measured, so we assume aluminum with $E = 69 \text{ GPa}$, $\rho = 2700 \text{ kg/m}^3$, $\nu = 0.33$, $h = 0.89 \text{ mm}$ and $L = 0.30 \text{ m}$, which gives a rough estimate for

$$D = \frac{(69 \times 10^9 \text{ Pa})(0.89 \times 10^{-3} \text{ m})^3}{12(1-0.33^2)} = 4.5 \text{ N} \cdot \text{m}$$

The bending wavelength is then found to be

$$\lambda_b = 2\pi \left(\frac{D}{\rho h}\right)^{\frac{1}{4}} \omega^{-\frac{1}{2}}$$

which falls from 0.20 m at 209 Hz to 0.070 m at 1774 Hz, which is why the patterns become finer as the frequency rises, with the bending waves travelling at 43 to 124 m/s.

The changes in bending wavelengths alongside derived natural resonant frequencies lead us to the observed phenomena. Grains at antinodes are thrown off every cycle while grains near nodes are barely moved, so sand drifts into the nodal lines and stays there; Zhou et al. (2016) showed that nodes act as attractors of the grain dynamics, with a dead zone at low amplitude caused by static friction.

III.

We used sand to visualize the vibration patterns on the plate. The sand was already in the box with the vibration plate and was very fine and white. We spread just enough sand to cover the plate and added more whenever needed. As the plate vibrated, the grains moved and collected in areas with little movement, making the patterns visible. The experiment was filmed in a room in the ITLL with all lights turned off and two flashlights held on opposite sides of the plate. The light illuminated the sand and plate so the patterns could be seen in the video.

IV.

We recorded this video in movie mode using a Canon EOS Rebel T7 digital DSLR with the exposure settings left on automatic. We do not have the exact aperture or shutter speed used during recording, and the ISO was also controlled automatically. The lens was roughly 33 cm from the plate. We used the widest view on the 18–55 mm zoom lens, so we estimate that the focal length was 18 mm. The choice to keep the view as wide as possible was made for the reason of recording the video easier. Holding the camera lower made it easier to hold, so the closer I could hold the camera to the plate, the less you would see in the video; this is why we chose the 18mm zoom. Based on this, the diagonal field of view was roughly 70°. The original video was recorded at 1920 × 1080 pixels and 25 fps, although looking back at the video, I wish we had increased the frame rate to capture the movement more smoothly. The camera can record up to about 30 fps at this resolution, or 50 or 60 fps at 1280 × 720 pixels, depending on the video system setting. There wasn't much image processing done, but we did make some editing choices in Google Vids. We added the frequencies next to the plate as they changed throughout the video so viewers could see which frequency produced each pattern. We also added a title at the beginning and music to accompany the video.

V.

The video reveals how sand forms different patterns as the vibration frequency changes. We like the contrast between the white sand and the black plate, which the lighting makes even stronger. The movement of the sand helps show how the plate's vibrations push the grains toward areas with less movement, making the standing-wave patterns visible. Although this experiment mainly demonstrates vibration and granular motion rather than typical fluid flow, the

video clearly shows how the sand responds to the vibrating surface. We fulfilled our intent by capturing different shapes at several frequencies. However, there is some camera shaking, and the transitions between frequencies could be cleaner. To improve the video, we would use a camera stand for stability and hold each frequency until the sand forms a clear pattern before changing it. During editing, we would trim unnecessary pauses while keeping enough footage to show how each pattern develops. To develop this idea further, we could test more frequencies and capture additional shapes.

References

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